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PARTIALLY ORDERED DOMAINS FOR REPRESENTING ACTIVITIES [1]

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Abstract: The paper describes structures which can be used to represent activities of broad class. The concept of event structure is generalized to represent activities which may be discrete, continuous, or of mixed nature. Configuration structures of the more general event structures are used to define axiomatically configuration domains. Elements of such domains are abstract representants of runs of represented activities. The partial order of elements reflects how each run extends to longer runs. It is shown that configuration domains define event structures which can be interpreted as interactions of sets of objects.

Keywords: Activity, run, event, causal dependency relation, conflict relation, event structure, configuration, configuration structure, partially ordered set, directed complete partially ordered set, configuration domain, transition, region.

CZĘŚCIOWO UPORZĄDKOWANE DZIEDZINY DO REPREZENTOWANIA DZIAŁALNOŚCI

Streszczenie: Praca opisuje struktury, których można użyć do reprezentowania szeroko rozumianych działalności. Uogólnia struktury zdarzeń tak, by mogły reprezentować działalności dyskretne, ciągłe i mieszanej natury. Konfiguracje tak uogólnionych struktur zdarzeń zostały użyte do aksjomatycznej definicji dziedzin konfiguracji. Elementy takich dziedzin są abstrakcyjnymi reprezentantami przebiegów reprezentowanych działalności. Pokazano, że dziedziny konfiguracji definiują struktury zdarzeń, które można interpretować jako współdziałania pewnych zbiorów obiektów.

Słowa kluczowe: Działalność, przebieg, zdarzenie, relacja zależności przyczynowej, relacja konfliktu, struktura zdarzeń, konfiguracja, struktura konfiguracji, zbiór częściowo uporządkowany, skierowany zupełny zbiór częściowo uporządkowany, dziedzina konfiguracji, tranzycja, region.

1 Introduction

The aim of the paper is twofold.

First, the concept of event structure such as in [6] is generalized in order to represent and relate activities that may be arbitrary combination of discrete and continuous behaviour (cf. [4]). Second, configuration structures of generalized event structures similar to those in [3] are used to define axiomatically a broad class of partially ordered sets of abstract elements which may represent runs of activities, to represent the partially ordered sets thus defined as generalized event structures, and to describe the properties of the respective structures.

Activities which consist of indivisible events can be described by specifying what events may occur, how each event depends on the events which occurred earlier, and how the events exclude each other in a run of the represented activity. The corresponding model is an *event structure* $\mathbf{E} = (E, \leq, \natural)$ consisting of a set E of events, a partial order $\leq$ on E, the *causal dependency relation*, and an irreflexive and symmetric relation $\natural$ on E, the *conflict relation*, such that: (1) the relations $\leq$ and $\natural$ are mutually exclusive, (2) every set $e^{\downarrow} = \{e' : e' \leq e\}$ is finite, and (3) for every $e, e', e'' \in E$ if $e \leq e'$ then the conflict $e' \natural e''$ is *inherited* from the conflict $e \natural e''$ in the sense that $e \natural e''$ implies $e' \natural e''$. If two events are not causally dependent nor in conflict then they are said to be *concurrent*.

Partial and full runs of the activity represented by an event structure $\mathbf{E} = (E, \leq, \natural)$ are represented by *configurations* of $\mathbf{E}$ where a configuration is a downwards-closed conflict-free subset x of E, i.e., a subset of E such that: (1) whenever $e \in x$ and $e' \leq e$ then $e' \in x$, and (2) for every $e, e' \in x$, it is not the case that $e \natural e'$. The set $C_{\mathbf{E}}$ of configurations of $\mathbf{E}$, partially ordered by inclusion, is a partially ordered set (a *poset*) $\mathbf{C_E}$. It is known (see [6]) that the poset $\mathbf{C_E} = (C_{\mathbf{E}}, \subseteq)$, called a *configuration structure*, is *coherent* (i.e. every $X \subseteq C_{\mathbf{E}}$ in which every pair of elements have an upper bound in $C_{\mathbf{E}}$ has the least upper bound $\bigcup X$ in $C_{\mathbf{E}}$), that it is a *prime algebraic domain* (i.e. every $c \in C_{\mathbf{E}}$ is the least upper bound of the set of those $d \subseteq c$ which are *complete prime* in the sense that $d \subseteq \bigcup X$ implies $d \subseteq x$ for some $x \in X$ for any set X for which $\bigcup X$ exists), and that it is *finitary* (i.e. its every complete prime is finite). Moreover, $d \in C_{\mathbf{E}}$ is a complete prime iff $d = \{e \in E : e \leq d\}$ and $e \natural e'$ iff there is no $d \in C_{\mathbf{E}}$ that contains e and e'.

Usually, events of an event structure are considered to represent actions. However, nothing prevents from considering them as situations and from dropping the requirement of finitely many predecessors of each event.

1.1. Example. Consider two independent objects v and w where the state of v at every moment t of the local time of v is represented by a number $g(t) \in [0, \infty)$ and the state of w at every moment s of the local time of w is represented by a number $h(s) \in [0, \infty)$.

Let F be the set of initial segments of functions $f : [0, \infty) \to [0, \infty)$ where an initial segment of f is the restriction of f to an initial segment of its domain including the empty segment. For every function $f \in F$, let $f^{\downarrow}$ denote the set of initial segments of f. Let $\mathbf{E}_1 = (E_1, \leq_1, \natural_1)$ consist of the set $E_1 = \{v\} \times F \cup \{w\} \times F$, of the least partial order $\leq_1$ such that $(v, g) \leq_1 (v, g')$ iff g is an initial segment of g' and $(w, h) \leq_1 (w, h')$ iff h is an initial segment of h', and of the relation $\natural_1$ that relates incomparable (v, g) and (v, g') and incomparable (w, h) and (w, h'). Then $\mathbf{E}_1$ can be regarded as an event structure even though its may have elements with infinitely many predecessors, and the sets $c_0 = \emptyset$, $c_1 = \{v\} \times g^{\downarrow}$, $c_2 = \{w\} \times h^{\downarrow}$, $c = \{v\} \times g^{\downarrow} \cup \{w\} \times h^{\downarrow}$ with $g, h \in F$ can be regarded as configurations of $\mathbf{E}_1$. $\sharp$

Consequently, every event structure $\mathbf{E} = (E, \leq, \natural)$ considered in this paper is supposed to consist of a set E of events, a partial order $\leq$ on E, the *causal dependency relation*, and an irreflexive and symmetric relation $\natural$ on E, the *conflict relation*, such that: (1) the relations $\leq$ and $\natural$ are mutually exclusive, and (2) for every $e, e', e'' \in E$ if $e \leq e'$ then the conflict $e' \natural e''$ is *inherited* from the conflict $e \natural e''$ in the sense that $e \natural e''$ implies $e' \natural e''$.

Configurations and related notions are defined as for standard event structures. If (c, c') is a pair of configurations such that $c \subseteq c'$ then it represents a *transition* $c \to c'$ from c to c'. Independence of transitions $t \to x$ and $t \to y$ is represented by the fact that the diagram $(x \leftarrow t \to y, x \to z \leftarrow y)$ is a *diamond* in the sense that t is the greatest lower bound of x and y and z is the least upper bound of x and y.

It is clear that configuration structures of the more general event structures need not to be finitary. However, their remaining properties are the same as the properties of configuration structures of standard event structures. Moreover, every such a structure $\mathbf{C} = (C, \to)$ enjoys the following properties:

(A1) $\mathbf{C}$ is a directed complete poset,

(A2) every nonempty subset X of C has the greatest lower bound,

(A3) for every diagram $(x \leftarrow t \to y, x \to z \leftarrow y)$ there exists a unique diamond $(x \leftarrow t' \to y, x \to z' \leftarrow y)$ such that $t \to t'$ and $z' \to z$, (A4) for every diamond $(x \leftarrow t \rightarrow y, x \rightarrow z \leftarrow y)$ with $x \rightarrow c$ or $y \rightarrow c$ there exists c' such that $c \rightarrow c'$ and $z \rightarrow c'$,

(A5) if $x \rightarrow z' \rightarrow z$ and $(x \leftarrow t \rightarrow y, x \rightarrow z \leftarrow y)$ is a diamond then there exists t' such that $t \rightarrow t' \rightarrow y$ and $(x \leftarrow t \rightarrow t', x \rightarrow z \leftarrow t')$ and $(z' \leftarrow t' \rightarrow y, z' \rightarrow z \leftarrow y)$ are diamonds,

(A6) if $t \rightarrow t' \rightarrow y$ and $(x \leftarrow t \rightarrow y, x \rightarrow z \leftarrow y)$ is a diamond then there exists z' such that $x \rightarrow z' \rightarrow z$ and $(x \leftarrow t \rightarrow t', x \rightarrow z \leftarrow t')$ and $(z' \leftarrow t' \rightarrow y, z' \rightarrow z \leftarrow y)$ are diamonds. $\sharp$

The properties (A1) - (A3) follow from the properties of event structures and from the definition of configurations. The property (A4) follows from the fact that events in conflict relation have no common upper bound. For (A5) it suffices to define t' as the greatest lower bound of z' and y and notice that a configuration structure is a subset of the distributive lattice of subsets of a set. Similarly, for (A6) it suffices to define z' as the least upper bound of t' and x.

In this paper we are interested in representing activities not only with the aid of the more general event structures and their configuration structures, but rather with the aid of arbitrary posets that enjoy only the relatively weak properties (A1) - (A6).

1.2. Example. $\perp \rightarrow a, b, c \rightarrow \top$ is a poset that enjoys the properties (A1) - (A6), but it is not an algebraic domain because $\top$ cannot be represented as the least upper bound of complete primes. Indeed, none of the elements a, b, c is a complete prime since it is dominated by the least upper bound of the remaining two elements and it is not dominated by any of them. $\sharp$

Posets which enjoy the properties (A1) - (A6) will be called configuration domains. It will be shown that every configuration domain defines a generalized event structure.

2 Configuration domains

The posets that are supposed to represent activities are defined as follows.

2.1. Definition. A *configuration domain* is a nonempty partially ordered set $\mathbf{C} = (C, \rightarrow)$ consisting of a set C of elements called *configurations* and of a partial order $\rightarrow$ in this set such that $\mathbf{C}$ enjoys the properties (A1) - (A6). $\sharp$

As in the case of configuration structures of event structures, elements of C are again supposed to represent runs of the represented activities. Ordered pairs of elements c and c' such that $c \rightarrow c'$ are called *transitions*.

Diagrams $(x \leftarrow t \rightarrow y, x \rightarrow z \leftarrow y)$ such that t is the greatest lower bound of x and y and z is the least upper bound of x and y are called *diamonds*. Given a diamond $D = (x \leftarrow t \rightarrow y, x \rightarrow z \leftarrow y)$, elements t, x, y, z are called *nodes* of D and transitions $t \rightarrow x$, $t \rightarrow y$, $x \rightarrow z$, $y \rightarrow z$ are called *sides* of D.

2.2. Example. Let $\mathbf{E}_1 = (E_1, \leq_1, \natural_1)$ be the event structure in example 1.1. The corresponding configuration domain is the partially ordered set $\mathbf{C}_1 = (C_1, \rightarrow_1)$ where C_1 is the set of configurations of $\mathbf{E}_1$ and $\rightarrow_1$ is the inclusion. Every diagram $(c \leftarrow_1 c \cap c' \rightarrow_1 c', c \rightarrow_1 c \cup c' \leftarrow_1 c')$ is a diamond in $\mathbf{C}_1$. In particular, the configurations $c_0 = \emptyset$, $c_1 = \{v\} \times g^{\downarrow}$, $c_2 = \{w\} \times h^{\downarrow}$, $c = \{v\} \times g^{\downarrow} \cup \{w\} \times h^{\downarrow}$ with $g, h \in F$ are elements of C_1 and the diagram $(c_1 \leftarrow_1 c_0 \rightarrow_1 c_2, c_1 \rightarrow_1 c \leftarrow_1 c_2)$ is a diamond. $\sharp$

2.3. Example. The poset in example 1.2 is a configuration domain $\mathbf{C}_2$. $\sharp$

The following propositions follow from the definition.

2.4. Proposition. $\mathbf{C}$ has the least element $\perp$. $\sharp$

2.5. Proposition. Every $c \in C$ has an upper bound which is a maximal element of $\mathbf{C}$. $\sharp$

2.6. Proposition. For every $c \in C$ the restriction of $\mathbf{C}$ to the set $c^{\downarrow} = \{d \in C : d \rightarrow c\}$ is a configuration domain (written also as $c^{\downarrow}$). $\sharp$

3 Independence and equivalence of transitions

The concept of a diamond can be used to define independence and equivalence of transitions. of a configuration domain.

3.1. Definition. If $(v \leftarrow u \rightarrow w, v \rightarrow u' \leftarrow w)$ is a diamond in a configuration domain $\mathbf{C} = (C, \rightarrow)$ then the transitions $u \rightarrow v$ and $u \rightarrow w$ are said to be *parallel independent*, and the transitions $u \rightarrow v$ and $v \rightarrow u'$, as well as the transitions $u \rightarrow w$ and $w \rightarrow u'$, are said to be *sequential independent* (cf. [2]). $\sharp$

3.2. Example. For the configuration domain $\mathbf{C}_1$ in example 2.2 and its configurations $c_0 = \emptyset$, $c_1 = \{v\} \times g^{\downarrow}$, $c_2 = \{w\} \times h^{\downarrow}$, $c = \{v\} \times g^{\downarrow} \cup \{w\} \times h^{\downarrow}$, the transitions $c_0 \rightarrow c_1$ and $c_0 \rightarrow c_2$ are parallel independent, and the transitions $c_0 \rightarrow_1 v$ and $c_1 \rightarrow_2 c$ are sequential independent. $\sharp$

3.3. Definition. By the *natural equivalence* of transitions of a configuration domain $\mathbf{C} = (C, \rightarrow)$ we mean the least equivalence relation $\equiv$ between transitions such that $u \rightarrow v \ \equiv \ w \rightarrow u'$ whenever in this configuration domain there exists a diamond $(v \leftarrow u \rightarrow w, v \rightarrow u' \leftarrow w)$. $\sharp$

3.4. Examples. Consider the configuration domain $\mathbf{C}_1$ in example 2.2. In this domain the transitions $c_0 \rightarrow c_1$ and $c_2 \rightarrow c$ are equivalent, and the transitions $c_0 \rightarrow c_2$ and $c_1 \rightarrow c$ are equivalent. $\sharp$

4 Regions of configuration domains

The existence in configuration domains of the natural equivalence of transitions makes it possible to adapt and exploit the concept of a region similar to that introduced in [1].

4.1. Definition. By a *region* of a configuration domain $\mathbf{C} = (C, \rightarrow)$ we mean a nonempty subset r of the set of elements of $\mathbf{C}$ such that:

$u \in r$ and $v \notin r$ and $w \rightarrow u' \ \equiv \ u \rightarrow v$ implies $w \in r$ and $u' \notin r$,
$u \notin r$ and $v \in r$ and $w \rightarrow u' \ \equiv \ u \rightarrow v$ implies $w \notin r$ and $u' \in r$. $\sharp$

4.2. Example. Given a function $f : [0, \infty) \rightarrow [0, \infty)$ and some $t \in [0, \infty) \cup \{\infty\}$, let f_t with $t \in [0, \infty)$ denote the restriction of f to the interval $[0, t]$ and let f_∞ denote f. In the configuration domain $\mathbf{C}_1$ in example 2.2 every set $c = \{v\} \times g^{\downarrow} \cup \{w\} \times h^{\downarrow}$, is a configuration, the sets $p(g, t) = \{v\} \times g_t^{\downarrow} \cup \{w\} \times F$ and $q(h, s) = \{v\} \times F \cup \{w\} \times h_s^{\downarrow}$ are minimal regions. $\sharp$

4.3. Example. In the configuration domain $\mathbf{C}_2$ in example 2.3 the sets $\{\bot, a\}$, $\{b, \top\}$, $\{\bot, b\}$, $\{a, \top\}$, $\{\bot, a\}$, $\{c, \top\}$, $\{\bot, c\}$, $\{a, \top\}$, $\{\bot, b\}$, $\{c, \top\}$, $\{\bot, c\}$, $\{b, \top\}$ are minimal regions. $\sharp$

From the definition of a region we obtain the following propositions.

4.4. Proposition. If $\mathbf{C} = (C, \to)$ is a configuration domain, r is a region of $\mathbf{C}$, and $(v \leftarrow u \to w, v \to u' \leftarrow w)$ is a diamond in $\mathbf{C}$, then $v \in r$ implies that $u \in r$ or $u' \in r$. $\sharp$

4.5. Proposition. The set of all configurations of $\mathbf{C}$ is a region of $\mathbf{C}$. $\sharp$

4.6. Proposition. If p and q are disjoint regions of $\mathbf{C}$ then $p \cup q$ is a region of $\mathbf{C}$. $\sharp$

4.7. Proposition. If p and q are different regions of $\mathbf{C}$ such that $p \subseteq q$ then $q - p$ is a region of $\mathbf{C}$. $\sharp$

Given a chain $(r_i : i \in I)$ of regions with $r = \bigcap(r_i : i \in I)$ and a transition $c \to d$ such that $c \in r$ and $d \notin r$, there exists $i_0 \in I$ such that $c \in r_i$ and $d \notin r_i$ for $i > i_0$. Consequently, for every transition $c' \to d'$ such that $c' \to d' \equiv c \to d$ we have $c \in r_i$ and $d \notin r_i$ for $i > i_0$, and thus $c \in r$ and $d \notin r$. Similarly, for $c \to d$ such that $c \notin r$ and $d \in r$ and for $c' \to d' \equiv c \to d$. So, r is a region. Hence, taking into account Kuratowski - Zorn Lemma, we obtain the following results.

4.8. Proposition. Every region of $\mathbf{C}$ contains a minimal region. $\sharp$

4.9. Proposition. Every configuration of $\mathbf{C}$ belongs to a minimal region. $\sharp$

4.10. Proposition. If a configuration s of $\mathbf{C}$ does not belong to a region r then there exists a minimal region r' such that $r \cap r' = \emptyset$ and $s \in r'$. $\sharp$

4.11. Proposition. Every region of $\mathbf{C}$ can be represented as a disjoint union of minimal regions. $\sharp$

Proof. Let r be a region of $\mathbf{C}$ that contains a minimal region m of $\mathbf{C}$. Then every minimal region n of $r - m$ is a minimal region of $\mathbf{C}$. Indeed, it cannot contain an element of m since then m could not be minimal. On the other hand, it cannot contain an element of $(r-m)-n$ since then m could be minimal. $\sharp$

4.12. Example. In the configuration domain $\mathbf{C}_1$ in example 2.2 we have the following decompositions of the set of configurations into disjoint union of minimal regions: $V = \{p(g,t) : t \in [0,\infty) \cup \{\infty\}, g : [0,\infty) \to [0,\infty)\}$ and $W = \{q(h,s) : s \in [0,\infty) \cup \{\infty\}, h : [0,\infty) \to [0,\infty)\}$. $\sharp$

4.13. Example. In the configuration domain $\mathbf{C}_2$ in example 2.3 we have the following decompositions of the set of configurations into disjoint union of minimal regions: $x = \{\{\bot, a\}, \{b, \top\}\}$, $y = \{\{\bot, a\}, \{c, \top\}\}$, $z = \{\{\bot, b\}, \{a, \top\}\}$, $t = \{\{\bot, b\}, \{c, \top\}\}$, $u = \{\{\bot, c\}, \{a, \top\}\}$, $v = \{\{\bot, c\}, \{b, \top\}\}$. $\sharp$

4.14. Proposition. For every element c of $\mathbf{C}$ and for every region r of $\mathbf{C}$ the subset $r|c = \{d \in r : d \to c\}$ of r is either empty or it is a region of $c^{\downarrow}$. $\sharp$

A proof follows from the fact that every diamond in $c^{\downarrow}$ is a diamond in $\mathbf{C}$.

4.15. Example. For the configuration domain $\mathbf{C}_1$ in example 2.2, its region $p(g,t) = \{v\} \times g_t^{\downarrow} \cup \{w\} \times F$, and its element $c = \{v\} \times g^{\downarrow} \cup \{w\} \times h^{\downarrow}$ we have $p(g,t)|c = \{v\} \times g_t^{\downarrow} \cup \{w\} \times h^{\downarrow}$, $q(h,s)|c = \{v\} \times g^{\downarrow} \cup \{w\} \times h_s^{\downarrow}$,
$V|c = \{p(g,t)|c : t \in [0,\infty) \cup \{\infty\}, g : [0,\infty) \to [0,\infty)\}$,
$W|c = \{q(h,s)|c : s \in [0,\infty) \cup \{\infty\}, h : [0,\infty) \to [0,\infty)\}$. $\sharp$

If c is a maximal element of $\mathbf{C}$ then due to (A4) every diamond with a side in $c^{\downarrow}$ is a diamond in $c^{\downarrow}$. Consequently, the maximal elements of $\mathbf{C}$ enjoy the following property.

4.16. Proposition. For every maximal element c of $\mathbf{C}$ and for every minimal region r of $\mathbf{C}$ the subset $r|c = \{d \in r : d \to c\}$ of r is either empty or it is a minimal region of $c^{\downarrow}$. $\sharp$

Note that if c is not a maximal element of $\mathbf{C}$ then the existence in $\mathbf{C}$ of a diamond with a side in $c^{\downarrow}$ does not necessarily implies the existence of such a diamond in $c^{\downarrow}$. Consequently, $r|c$ need not be a minimal region of $c^{\downarrow}$ even though the region r is a minimal region of $\mathbf{C}$.

5 Configuration domains as event structures

Given a configuration domain $\mathbf{C} = (C, \rightarrow)$, we can assign to $\mathbf{C}$ an event structure
$\mathbf{E_C} = (E_\mathbf{C}, \leq_\mathbf{C}, \natural_\mathbf{C})$. To this end we introduce a partial order $\preceq$ on the set of minimal regions of $\mathbf{C}$ and we define $\mathbf{E_C}$ as follows.

Let $R_\mathbf{C}$ denote the set of minimal regions of $\mathbf{C}$. Let $D_\mathbf{C}$ denote the set of decompositions of the set of elements of $E_\mathbf{C}$ into disjoint unions of minimal regions, every decomposition defined as a set d of mutually disjoint minimal regions from $R_\mathbf{C}$ such that $\bigcup d = C$.

The underlying set $E_\mathbf{C}$ of $\mathbf{E_C}$ is defined as the set of pairs (d, r) consisting of a decomposition $d \in D_\mathbf{C}$ and of a minimal region $r \in d$.

The partial order $\leq_\mathbf{C}$ is defined as the least partial order such that $(d, r) \leq_\mathbf{C} (d', r')$ if $r \preceq r'$ and $r \neq r'$ or if $d = d'$ and $r = r'$.

The conflict relation $\natural_\mathbf{C}$ is defined by assuming $(d, r) \natural_\mathbf{C} (d', r')$ iff there is no $c \in C$ such that $r|c$ and $r'|c$ are minimal regions in $c^{\downarrow}$.

Now we are going to prove that $\mathbf{E_C}$ is indeed an event structure.

The partial order $\preceq$ between minimal regions of $\mathbf{C}$ can be introduced as follows.

5.1. Definition. Given $x, y \in R_\mathbf{C}$, we write $x \preceq y$ iff for every $v \in y$ there exists $u \in x$ such that $u \rightarrow v$, for every $u \in x$ there exists $v \in y$ such that $u \rightarrow v$, and the following conditions are satisfied:

(1) $t \in x$ iff $w \in y$, for every diamond $(u \leftarrow t \rightarrow w, u \rightarrow v \leftarrow w)$ with $u \in x$ and $v \in y$,

(2) $t' \in x$ iff $w' \in y$, for every diamond $(t' \leftarrow u \rightarrow v, t' \rightarrow w' \leftarrow v)$ with $u \in x$ and $v \in y$. $\sharp$

5.2. Proposition. The relation $\preceq$ is a partial order on $R_\mathbf{C}$. $\sharp$

For a proof it suffices to notice that the relation $\preceq$ follows the partial order in $\mathbf{C}$.

The construction of the event structure $\mathbf{E_C} = (E_\mathbf{C}, \leq_\mathbf{C}, \natural_\mathbf{C})$ is based on the following observations (cf. [5]).

First, the properties (A5) and (A6) imply an important property of minimal regions.

5.3. Proposition. Every minimal region r is *convex* in the sense that $w \in r$ for every w such that $u \to w \to v$ for some $u \in r$ and $v \in r$. $\sharp$

Second, minimal regions wich are not disjoint are incomparable with respect to the partial order $\preceq$.

5.4. Proposition. If minimal regions $x, y \in R_\mathbf{C}$ are not disjoint and different then neither $x \preceq y$ nor $y \preceq x$. $\sharp$

Proof. Suppose that x and y are different minimal regions of $R_\mathbf{C}$ such that $x \cap y \neq \emptyset$. Then $x - y$ and $y - x$ are nonempty and there exist $u \in x - y$, $v \in y - x$, and $w, z \in x \cap y$ such that u and w are adjacent nodes of a diamond U, z and v are adjacent nodes of a diamond V, and the nodes of the diamond $W = (w \leftarrow w \wedge z \to z, w \to w \vee z \leftarrow z)$ are in $x \cap y$.

Consider the case in which $w = u \vee u'$ for some u' not in x and $z = v \wedge v'$ for some v' not in y. Then $u' \in y$, $v' \in x$, and the condition (1) is not satisfied for $z \to v$ and the diamond $(v \leftarrow z \to v', v \to v \vee v' \leftarrow v')$. Consequently, $x \preceq y$ does not hold.

Similarly, in the other possible cases we come to the conclusion that neither $x \preceq y$ nor $y \preceq x$. $\sharp$

Third, in some configuration domains all disjoint minimal regions are comparable with respect to the partial order $\preceq$.

5.5. Proposition. If every elements a and b of $\mathbf{C}$ have the least upper bound $a \vee b$ and minimal regions $x, y \in R_\mathbf{C}$ are disjoint then either $x \preceq y$ or $y \preceq x$. $\sharp$

Proof. It is impossible that u and v are incomparable for all $u \in x$ and $v \in y$ since one of the regions x or y contains $u \vee v$ or $u \vee v$.

Suppose that $u \to v$ for $u \in x$ and $v \in y$. As x and y are disjoint and convex, it suffices to prove that every element of y has a predecessor in x. Consider $w \in y$. If $v \to w$ then $u \to w$. If $w \to v$ then $u' \to w$ for $u' = u \wedge w$ and by considering the diamond $(u' \leftarrow u \to v, u' \to w \leftarrow v)$ we obtain that

$u' \in x$. If w and v are incomparable then either $v \wedge w \in y$ and we may replace w by $v \wedge w$ and proceed as in the previous case, or $v \vee w \in y$ and by considering the diamond $(u' \leftarrow u \rightarrow w, u \rightarrow v \vee w \leftarrow w)$ we obtain that $u' \rightarrow w$ for $u' \in x$. On the other hand, $u \rightarrow v$ for $u \in x$ and $v \in y$ excludes $v' \rightarrow u'$ for $u' \in x$ and $v' \in y$ since x and y are convex. Hence $x \preceq y$.

Similarly, in the case $v \rightarrow u$ we obtain $y \preceq x$. $\sharp$

One of the consequences of these observations is the following proposition.

5.6. Proposition. For every $d \in D_{\mathbf{C}}$ the subset $\{d\} \times \{r \in R_{\mathbf{C}} : r \in d\}$ of $E_{\mathbf{C}}$ is a maximal chain. $\sharp$

A proof follows from the fact that for every maximal $c \in C$ the restrictions $r|c$ of r from d form a decomposition of $R_{c^{\downarrow}}$ into a disjoint union of minimal regions (see proposition 4.16).

5.7. Proposition. For every $c \in \mathbf{C}$, the subset $X_c = \{(d, r) :\in E_{\mathbf{C}} : c \in r\}$ of $E_{\mathbf{C}}$ is a maximal conflict-free antichain of $\mathbf{E_C}$. $\sharp$

A proof follows from the fact that no minimal regions in X_c are disjoint and from the fact that c belongs to one minimal region of every decomposition $d \in D_{\mathbf{C}}$.

5.8. Proposition. If $(d, r) \leq_{\mathbf{C}} (d', r')$ then $(d, r) \natural_{\mathbf{C}} (d', r')$ does not hold. $\sharp$

For a proof it suffices to take into account propositions 2.5 and 4.16 and notice that $r|c$ and $r'|c$ are minimal regions of $c^{\downarrow}$ for a maximal c that contains the minimal region $r|c$.

These results can be summarized in the following theorem.

5.9. Theorem. Given a configuration domain $\mathbf{C} = (C, \rightarrow)$, the system $\mathbf{E_C} = (E_{\mathbf{C}}, \leq_{\mathbf{C}}, \natural_{\mathbf{C}})$ is an event structure with configurations $X_c^{\downarrow} = \{(d, r) : (d, r) \leq_{\mathbf{C}} (d', r') \text{ for some } (d', r') \in X_c\}$ corresponding to maximal conflict-free antichains $X_c = \{(d, r) :\in E_{\mathbf{C}} : c \in r\}$ of $\mathbf{E_C}$, and with maximal chains $\{d\} \times \{r \in R_{\mathbf{C}} : r \in d\}$. $\sharp$

Each decomposition $d \in D_{\mathbf{C}}$ can be interpreted as an indivisible object. Each minimal region $r \in d$ can be interpreted as an occurrence of this object. Consequently, the event structure $\mathbf{E_C}$ can be interpreted as an interaction of a set of indivisible objects.

5.10. Example. For the configuration domain $\mathbf{C} = \mathbf{C}_1$ in example 2.2 the structure $\mathbf{E_C}$ is isomorphic to the structure $\mathbf{E}_1$ in example 1.1 with the isomorphism i such that $i(v, g_t) = (V, p(g, t))$ with $g_t = g|[0, t]$ for $g : [0, \infty) \to [0, \infty)$ and $i(w, h_s) = (W, q(h, s))$ with $h_s = h|[0, s]$ for $h : [0, \infty) \to [0, \infty)$. $\sharp$

5.11. Example. For the configuration domain $\mathbf{C} = \mathbf{C}_2$ in example 2.3 the structure $\mathbf{E_C}$ consists of independent chains $(x, \{\bot, a\}) \leq_{\mathbf{C}} (x, \{b, \top\})$,
$(y, \{\{\bot, a\}) \leq_{\mathbf{C}} (y, \{c, \top\})$,
$(z, \{\bot, b\}) \leq_{\mathbf{C}} (z, \{a, \top\})$, $(t, \{\bot, b\}) \leq_{\mathbf{C}} (t, \{c, \top\})$, $(u, \{\bot, c\}) \leq_{\mathbf{C}} (u, \{a, \top\})$,
$(v, \{\bot, c\}) \leq_{\mathbf{C}} (v, \{b, \top\}$.
and of the empty conflict relation. $\sharp$

6 Concluding remarks

Making use of the fact that events of event structures can be interpreted as situations and that they need not to have only finitely many predecessors, we have generalized the concept of event structure and the concept of configuration structure. Then, using only some properties of configuration structures as axioms, we have introduced the concept of configuration domains. Finally, we have shown that configuration domains define event structures which represent interactions of sets of indivisible objects. Thus new structures have been introduced which can be used to represent in the same form and relate activities of a broad class.

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