THE INTEGRATION OF THE MINOR PLANET (OR COMET) MOTION EQUATIONS

I. Perturbations due to Jupiter and Saturn *

by

KRZYSZTOF ZIOŁKOWSKI

The general description of the program of integration of the minor planet or comet motion equations, taking into account the perturbational influence of the greatest planets (Jupiter and Saturn), is presented in the paper. The program was performed for the URAL-2 electronic digital computer in the Computation Center of the Polish Academy of Sciences.

§ 1. Minor planet (or comet) motion under the influence of the gravitational force of the Sun, disturbed by the gravitation of great planets, is given by a set of the three ordinary differential equations of the second order in the heliocentric, rectangular coordinate system x, y, z [1]:

$$\begin{aligned} \frac{d^2x}{dt^2} &= -k^2(M+m)\frac{x}{r^3}+k^2\sum_i m_i\left(\frac{x_i-x}{\Delta_i^3}-\frac{x_i}{r_i^3}\right), \\ \frac{d^2y}{dt^2} &= -k^2(M+m)\frac{y}{r^3}+k^2\sum_i m_i\left(\frac{y_i-y}{\Delta_i^3}-\frac{y_i}{r_i^3}\right), \\ \frac{d^2z}{dt^2} &= -k^2(M+m)\frac{z}{r^3}+k^2\sum_i m_i\left(\frac{z_i-z}{\Delta_i^3}-\frac{z_i}{r_i^3}\right), \end{aligned} \tag{1}$$

where the following designations are used:

t — time;

x, y, z — coordinates of a minor planet;

x_i, y_i, z_i — coordinates of an i-th planet that disturbes motion;

$r = (x^2+y^2+z^2)^{1/2}$ — distance from a minor planet to the Sun;

$r_i = (x_i^2+y_i^2+z_i^2)^{1/2}$ — distance from an i-th disturbing planet to the Sun;

$\Delta_i = [(x-x_i)^2+(y-y_i)^2+(z-z_i)^2]^{1/2}$ — distance from a minor planet to an i-th disturbing planet;

---

* The paper was submitted on March 1965.

M — mass of the Sun;
m — mass of the minor planet;
m_i — mass of an i-th planet disturbing a motion;
k — Gauss constant.

The mass m of planetoid is much less than that of the Sun M, so taking the last one as a unit, one receives the following form of the equations:

$$\frac{d^2x}{dt^2} = -k^2\frac{x}{r^3} + k^2\sum_i m_i\left(\frac{x_i - x}{\Delta_i^3} - \frac{x_i}{r_i^3}\right),$$

$$\frac{d^2y}{dt^2} = -k^2\frac{y}{r^3} + k^2\sum_i m_i\left(\frac{y_i - y}{\Delta_i^3} - \frac{y_i}{r_i^3}\right), \tag{2}$$

$$\frac{d^2z}{dt^2} = -k^2\frac{z}{r^3} + k^2\sum_i m_i\left(\frac{z_i - z}{\Delta_i^3} - \frac{z_i}{r_i^3}\right).$$

The solution of the (2) set of equations, with the given initial conditions in the instant t_0, enables the determination of a position and a velocity of a minor planet in the given instant t_k. In the astronomical practice it reduces to looking for, so called, osculating orbital elements in the epoque t_k on the base of the known osculating elements of the initial epoque t_0.

§ 2. A transition from the elliptic orbit elements to the heliocentric, ecliptic, rectangular coordinates and their first derivatives with respect to time, that is to the respective velocity components of a body on an orbit, is described by the set of the following formulas [1]:

$$x = A_x(\cos E - e) + B_x \sin E,$$

$$y = A_y(\cos E - e) + B_y \sin E,$$

$$z = A_z(\cos E - e) + B_z \sin E,$$

$$\frac{dx}{dt} = \frac{k}{r\sqrt{a}}(-A_x \sin E + B_x \cos E), \tag{3}$$

$$\frac{dy}{dt} = \frac{k}{r\sqrt{a}}(-A_y \sin E + B_y \cos E),$$

$$\frac{dz}{dt} = \frac{k}{r\sqrt{a}}(-A_z \sin E + B_z \cos E),$$

where the coefficients A and B are given by the following formulas:

$$A_x = a(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i),$$

$$A_y = a(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i),$$

$$A_z = a\sin\omega\sin i,$$

$$B_x = a\sqrt{1-e^2}(-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i),$$

$$B_y = a\sqrt{1-e^2}(-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i),$$

$$B_z = a\sqrt{1-e^2}\cos\omega\sin i,$$

where the following denotations are used:

e — eccentricity of orbit;

a — semi-major axis of orbit;

Ω — longitude of ascending node;

ω — longitude of perihelion in orbit;

i — inclination of orbit plane to the plane of ecliptic;

E — eccentric anomaly related to an arbitrary instant t by the Kepler equation:

$$E - e\sin E = n(t-T) = M,$$

where

$$n = \frac{k}{a^{3/2}}$$

is the mean diurnal motion and T denotes the moment of perihelion passage. $M = n(t-T)$ is called mean anomaly. Kepler's equation can be solved with respect to E, e.g., by the Newton method of successive approximations.

Passing from the coordinates and the respective velocity components to the elliptic elements can be done using the following formulas [1]:

$$a = \frac{r}{2-rV^2},$$

$$E = \operatorname{arctg}\frac{R}{\sqrt{a}\,(rV^2-1)},$$

$$e = \frac{rV^2-1}{\cos E},$$

$$\omega = \operatorname{arctg}\frac{P_z}{Q_z}, \tag{4}$$

$$\Omega = \operatorname{arctg}\frac{P_y\cos\omega - Q_y\sin\omega}{P_x\cos\omega - Q_x\sin\omega},$$

$$i = \operatorname{arctg}\frac{-P_z\sin\Omega}{\sin\omega(P_x\sin\omega + Q_x\cos\omega)},$$

where the following denotations are used:

$$V^2 = \frac{1}{k^2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right],$$

$$R = \frac{1}{k}\left(x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}\right),$$

and:

$$P_x = \frac{x}{r}\cos E - \frac{dx}{dt}\frac{\sqrt{a}}{k}\sin E,$$

$$P_y = \frac{y}{r}\cos E - \frac{dy}{dt}\frac{\sqrt{a}}{k}\sin E,$$

$$P_z = \frac{z}{r}\cos E - \frac{dz}{dt}\frac{\sqrt{a}}{k}\sin E,$$

$$Q_x = \frac{1}{\sqrt{1-e^2}}\left[\frac{x}{r}\sin E + \frac{dx}{dt}\frac{\sqrt{a}}{k}(\cos E - e)\right],$$

$$Q_y = \frac{1}{\sqrt{1-e^2}}\left[\frac{y}{r}\sin E + \frac{dy}{dt}\frac{\sqrt{a}}{k}(\cos E - e)\right],$$

$$Q_z = \frac{1}{\sqrt{1-e^2}}\left[\frac{z}{r}\sin E + \frac{dz}{dt}\frac{\sqrt{a}}{k}(\cos E - e)\right].$$

§ 3. We cannot give the analitical solutions of the (2) equations set, so the only way to solve them is numerical integration. The Runge-Kutta method, modified and adopted to the computations on electronic digital computers by Gill [2], was chosen from many different methods of numerical integration of ordinary differential equations. The choice was made on the basis of conclusions given in [7]. The characteristics of Runge-Kutta-Gill method are the following:

a) There is a possibility of setting of a simple and short computation program for a digital computer.

b) To begin computations it is necessary to know only one starting point, that is, simply initial conditions in contrast with commonly used in astronomical practice difference method of integration of equations.

c) Like in point b) to find the sought functions in the $(k+1)$—th step it is necessary to know the values of these functions in the k—th, that is the previous step.

d) For the sake of b) and c) it is easy to change the value of step of integration during calculations without necessity of doing additional calculations.

e) Relatively small accumulation of a total integration error [7].

In the program of integration of the (2) equations described in the paper the Runge-Kutta-Gill method of the 4-th order is used. It corresponds to throwing away in the corresponding expansions the terms of the order of the 5-th power of step of integration. The use of this method requires a change of the system (2) of three differential equations of the second order into the system of seven differential equations of the first order. If the set (2) has the general form:

$$\frac{d^2 x_n}{dt^2} = f_n(x_1, \ldots, x_n), \quad n = 1, 2, 3, \tag{5}$$

where $x_1 = x, x_2 = y, x_3 = z$ in the previous notation, then substituting:

$$\frac{dx_n}{dt} = z_n, \quad n = 1, 2, 3,$$

one obtain the equivalent system of differential equations of the first order:

$$\frac{dy_n}{dt} = g_n(y_1, \ldots, y_m), \quad m = 1, 2, \ldots, 7, \tag{6}$$

where:

$$\begin{aligned} y_1 &= t, & g_1 &= 1, \\ y_2 &= z_1, & g_2 &= f_1, \\ y_3 &= z_2, & g_3 &= f_2, \\ y_4 &= z_3, & g_4 &= f_3, \\ y_5 &= x_1, & g_5 &= z_1, \\ y_6 &= x_2, & g_6 &= z_2, \\ y_7 &= x_3, & g_7 &= z_3. \end{aligned} \tag{7}$$

It is equivalent to the set (5) of differential equations of the second order

Next, if we introduce the following denotations:

$$\frac{dy_{m,l}}{dt} = g_m(y_{1,l-1}, y_{2,l-1}, \ldots, y_{7,l-1}) = K_{m,l},$$

where the index l in the 4-th order method for each k takes successively the values 1, 2, 3 and 4 then the general formula for the values of the sought functions in the k-th step of integration, if the $(k-1)$-th step is known, has the form [2]:

$$\begin{gathered} y_{m,l} = y_{m,l-1} + h[a_l(K_{m,l} - b_l q_{m,l-1})], \\ m = 1, 2, \ldots, 7; \quad l = 1, 2, 3, 4, \end{gathered} \tag{8}$$

where h denotes step of integration and:

$$q_{m,l} = q_{m,l-1} + 3[a_l(K_{m,l} - b_l q_{m,l-1})] - c_l K_{m,l},$$

and the constants a, b and c have the following values:

$$\begin{aligned} a_1 &= 1/2, & b_1 &= 2, & c_1 &= 1/2, \\ a_2 &= 1-\sqrt{1/2}, & b_2 &= 1, & c_2 &= 1-\sqrt{1/2}, \\ a_3 &= 1+\sqrt{1/2}, & b_3 &= 1, & c_3 &= 1+\sqrt{1/2}, \\ a_4 &= 1/6, & b_4 &= 2, & c_4 &= 1/2. \end{aligned}$$

It is assumed that the subsidiary coefficients q fulfill the conditions:

$$q_{m,1}(t_0) = 0,$$

$$q_{m,1}(t_{k+1}) = q_{m,4}(t_k),$$

where $t_k = t_0 + kh$.

The only imperfection of the accepted method of numerical integration of the equations (2), that is important here, is the necessity of fourfold calculation of the right-hand members of equations in each step of integration.

§ 4. Since the discussed program of integration of the minor planet motion equation, taking into consideration the perturbations due to Jupiter and Saturn, will be the part of the total program of minor planets (or comets) orbit calculation, it consists of several subroutines. These are the following ones:

a) Changing of orbit elements to corresponding coordinates and velocity components and vice versa.

b) Integration of the equations (6) by the Runge-Kutta-Gill method.

c) Calculation of the right-hand members of the equations (6).

The first subroutine is used to realize calculations given according to the need by the formulas (3) or (4). The second subroutine is used to calculations during one step of integration according to the formula (8). Since both the subroutines are quite simple we omit here their detail description. But we will consider the most important part of the discussed program, the subroutine of calculation of the right-hand members of the equations (6).

As it is seen from the equivalence (7), the problem of calculation of the right-hand members of the equations (6) reduces to calculation of the right-hand members of the equations (5). The main difficulty is a consequence of the fact that the right-hand members of the equations (5) are the functions of the coordinates of planets, disturbing the motion of a considered body. These coordinates are the functions of time of course. The values of these coordinates can be achieved by the different ways, e.g.:

a) To integrate the equations of the motion of particular planets simultaneously with the equations (2).

b) On the base of theory of planet motion.

c) Interpolating the planet coordinates tables put into computer.

d) By various approximate methods on the base of the fact, that the planets orbits have the shapes similar to ellipses with small eccentricities.

All these methods have various inconveniences. The first method considerably prolongs the whole calculation process, the second one increases, moreover, to a high degree the program for a computer. The third method requires putting into a computer the necessary part of the tables that takes a great number of memory places. Moreover, the approximate methods not always admit to obtain the required accuracy of the coordinates by the relatively simple method.

Taking into account difficulties in generating the disturbing planets coordinates in a computer and the fact that perturbations in minor planets and short period comets motions are caused mostly by the greatest planets Jupiter and Saturn, in the first part of the paper the influence of these two planets only is considered. The second part will be devoted to the discussion of the way of calculation of perturbations due to all remaining great planets. Taking into account, in the first approximation, the perturbations connected with Jupiter and Saturn only is justified by the fact that in the majority of orbital problems the errors of numerical integration of the equations (2), issued from the input elements inaccuracy and from the errors of integration method, are of the same order as the values of remaining planets perturbations or even sometimes exeed them.

The Jupiter and Saturn coordinates, required for each instant t_k from the integration interval, are obtained in the discussed program by interpolation of their values from the part of tables put beforehand into a computer and necessary for the given integration interval. The tables were described in the paper [8]. The choice of this way of generation of the disturbing planets coordinates was inspired by:

a) The necessity of taking into account the full accuracy of Jupiter and Saturn coordinates because of their important role in the perturbation term of the right-hand members of the equations (2).

b) Much shorter time needed to obtain them in comparison with the other methods.

The Jupiter and Saturn coordinates tables mentioned above are used in the program according to the general method given in the paper [3]. We will discuss its basic idea in application to the program.

Let t denotes the instant for which the Jupiter and Saturn coordinates, expressed in Julian Days from 2390000.5 (the beginning of the tables) [8], are necessary. It can take the following values: $t_k = T_p + kh$ where h is a step of integration and if $h > 0$ then $p = 1$ and if $h < 0$ then $p = 2$; k takes the values being the natural numbers $0, 1, 2, \ldots$; T_1 and T_2 can be considered as the beginning and the end of the time interval from which the Jupiter and Saturn coordinates will be needed.

The six-point Lagrange formula was used to the interpolation. According to it the arbitrary coordinate $x_i(t)$, where i denotes its number according to the table:

i	1	2	3	4	5	6
coordinate	x	y	z	x	y	z
	Jupiter			Saturn		

is given by the formula:

$$x_i(t) = \sum_{j=-2}^{3} L_j x_i(t_j), \tag{9}$$

where L_j are the respective Lagrange interpolation coefficients and:

$$t_j = 40\left[\text{entier}\left(\frac{t}{40}\right) + j\right].$$

Because it is fixed that the initial elements are given for the epoque for which the number of days is integer and the step of integration can be equal to the number of days, i e., integer and even (2, 4, 6, etc.) (even — because of the necessity of dividing it into two in the Runge-Kutta-Gill method) and because the interval of the tables used is 40 days [8] it is necessary to put into a computer (or to calculate if it is necessary, but it prolongs greatly the interpolation time) $120 = 6 \times 20$ Lagrange interpolation coefficients. They are placed in the operational memory beginning from the address L in the succession of increasing arguments (0, 0.025, 0.050, ..., 0.500) and for the given argument in the succession of increasing j.

Next, it is necessary to put into a computer memory the part of the tables that corresponds to T_1 and T_2. Only the full zones are put into a memory [8] and the numbers denoted the first and the last one among the necessary ones are given by the formulas:

$$s_1 = \text{entier}\left(\frac{T_1 - 80}{2000}\right), \quad s_2 = \text{entier}\left(\frac{T_2 + 119}{2000}\right).$$

Because the necessary part of the tables often does not hold in the operational memory it is put into the external memory (in our case it is the drum memory) and in a case of need the smaller parts of the needed groups are brought to the ferrite memory. On the drum the tables are placed in groups of 150 numbers, beginning from the address A, in the following succession:

A	group 0,	zone s_1,
. . . . .		
$A+149$		
$A+150$	checking sum of the group 0	
$A+151$	group 1	
. . . . . .		
$A+301$		
$A+302$	checking sum of the group 1	
. . . . . .		
. . . . . .		
. . . . . .		

$B = A + 151\,R$ checking sum of the group R, where $R = 2(s_2 - s_1 + 1)$ is the number of the last group. The checking sum assures the correctness of passing of the respective groups from a drum to the operational memory and vice versa. In the ferrite memory there are always two groups placed from the address a to $b = a + 299$: the first group from $a + 150(p-1)$ to $a + 150(p-1) + 149$ and the second group from $a + 150(2-p)$ to $a + 150(2-p) + 149$.

In order to examine whether the addresses of coordinates for the argument t_j for some instant t are available in the operational memory, i.e., whether they are greater than $a + 149$ while $h > 0$ and not greater than $a + 149$ while $h < 0$ it is enough to check the inequality [3]:

$$(3-2p)\{t - 2000 s_1 - [v_p + (3-2p)(r-p+1)]1000\} < 40(p+1) \qquad (10)$$

with the assumption that the last group put into the ferrite memory was the one that begins on the drum from the address:

$$A + 151[v_p + r(3-2p)], \quad r = 0, 1, 2, \ldots, R,$$

where:

$$v_p = \text{entier}\left(\frac{T_p - 2000 s_1 + 199p - 279}{1000}\right)$$

denotes the number of the group that will be put into the operational memory as the first one (for $h > 0$ v_p takes the values 0 or 1; for $h < 0$ R or $R-1$).

If the inequality (10) is fulfilled, it means that the coordinates $x_i(t_j)$ are in the operational memory and their addresses are given by the formula:

$$\langle x_i(t_j)\rangle = a + 299(p-1) + 5(2-p) + 6[j + (3-2p)w_p] + (i-6) \qquad (11)$$

$$i = 1, 2, 3, 4, 5, 6; \quad j = -2, -1, 0, 1, 2, 3,$$

where:

$$w_p = \text{entier}\left(\left|\frac{t - 2000 s_1 - [v_p + (r-p+1)(3-2p)]1000}{40}\right|\right)$$

is the integer, smaller than 50, determining in which of the successive hexads of coordinates corresponding to one date there is the necessary coordinate (in the ferrite memory there are always 50 hexads).

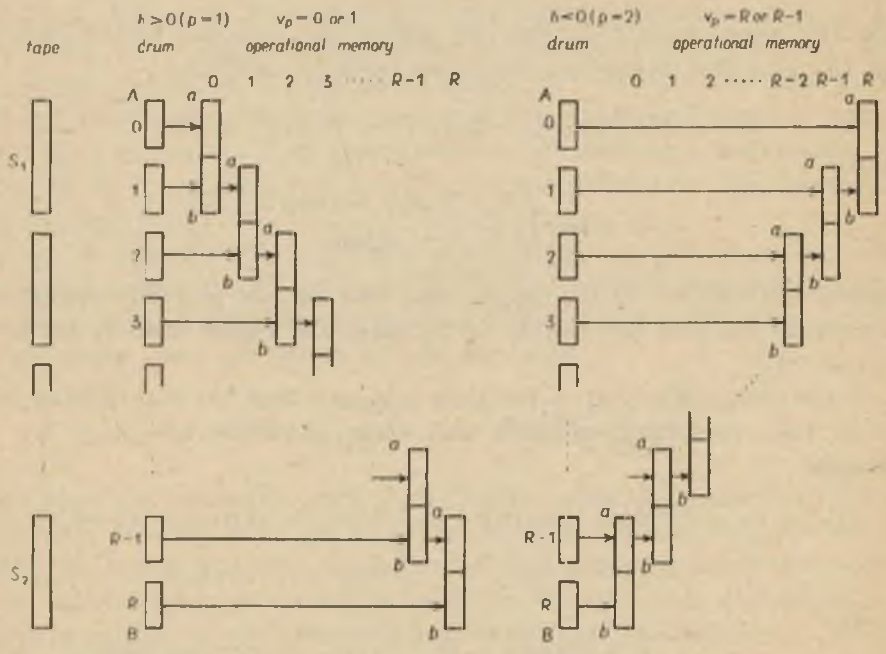
If the inequality (10) is not fulfilled then the translocation of groups in the operational memory is made. It is presented schematically at the figure, p. 98. More exactly: the group beginning from the address $a + 150(2-p)$ is put in the place of the group beginning from $a + 150(p-1)$ and the group from the drum with the beginning address $A + 151[v_p + (r+1) \times (3-2p)]$ is put in the vacant place. As a result of this process the necessary coordinates have been placed in the operational memory and their addresses are defined, as previously, by the formula (11).

The address of the corresponding Lagrange Interpolation coefficients will be found from the formula:

$$\langle L_j \rangle = L + 6(t - t_{j=0} - 1) + (j+2).$$

Knowing the addresses of the actual quantities, occurring in the Lagrange interpolation formula (9), it is easy to find the sought values of the Jupiter and Saturn coordinates for given instant t that are necessary when calculating the right-hand members of the equations (5).

Although the Runge-Kutta-Gill method requires the fourfold calculation of the right-hand members of the equations in each step of integration as it is mentioned above, taking advantage from the fact, that it



is performed for instants $t + h/2$ and $t + h$ only, it is enough to interpolate the coordinates of the great planets only twice. Each intepolation follows the checking of the inequality (10). It is necessary to assure the presence of the values of perturbating planets coordinates for the corresponding instants in the operational memory.

The described subroutines together with the necessary standard subroutines and working places take about 65 % of the operational memory of the URAL-2 computer (2048 40-bit words). The program is written in the KLIPA language. The organization of the input and output of the discussed program will be given in the next part.

§ 5. To check the correctness of the operation of the discussed program of integration of the minor planet (comet) motion equations by the Runge-Kutta-Gill method, the short period Grigg-Skjellerup comet has been used. The theory of the comet motion has been worked out by G. Sitarski ([4], [5], [6]). It is the comet of the Jupiter group with the following orbit elements (like in [4] they are given for one epoque only to give the general view):

Epoch and osculation: 1952 March 9.0 ET

$$
\begin{array}{llll}
T = 1952 \text{ March } 11.155905 \text{ ET} & \Omega = 215^\circ 22' 58''.44 & \multirow{4}{*}{} \\
M = 359^\circ 34' 0''.30 & \omega = 356\ 21\ 27.68 & \\
\varphi = \ \ 44\ 42\ 59.83 & \pi = 211\ 44\ 26.12 & 1950.0 \\
n = 723''.45490 & i = \ \ 17\ 37\ 40.42 &
\end{array}
$$

where φ is eccentricity angle ($e = \sin\varphi$), and π — longitude of perihelion ($\pi = \Omega + \omega$). The denotations of the remaining elements are as previously.

Table 1

Method	δM	$\delta\varphi$	$\delta\Omega$	$\delta\pi$	δi	δn
	1952 Mar. 9.0 — 1947 Apr. 5.0 starting elements S_0' [4]					
GS	$-1697''.00$	$+172''.84$	$-26''.29$	$+86''.86$	$+69''.73$	$+0''.98015$
KZ	-1697.15	$+172.85$	-26.28	$+86.89$	$+69.84$	$+0.98023$
GS-KZ	$+0.15$	-0.01	-0.01	-0.03	-0.11	-0.00008
	1952 Mar. 9.0 — 1957 Jan. 2.0 starting elements S_0' [4]					
GS	$+1218.28$	$+16.73$	$+22.00$	-89.58	$+28.74$	$+0.20255$
KZ	$+1218.77$	$+16.90$	$+22.06$	-89.53	$+28.78$	$+0.20317$
GS-KZ	-0.49	-0.17	-0.06	-0.05	-0.04	-0.00062
	1957 Jan. 2.0 — 1962 Jan. 16.0 starting elements S_1' [4]					
GS	-1435.96	-198.50	-88.50	$+111.75$	-55.30	-0.63346
KZ	-1436.13	-198.49	-88.56	$+111.71$	-55.34	-0.63363
GS-KZ	$+0.17$	-0.01	$+0.06$	$+0.04$	$+0.04$	$+0.00017$
	1962 Jan. 16.0 — 1963 Sept. 8.0 starting elements S_2 [5]					
GS	-1429.75	$+988.89$	-2304.75	-374.64	$+1454.31$	$+0.06155$
KZ	-1429.65	$+989.13$	-2304.89	-374.70	$+1454.52$	$+0.06185$
GS-KZ	-0.10	-0.24	$+0.14$	$+0.06$	-0.21	-0.00030
	1963 Sept. 8.0 — 1964 Sept. 22.0 starting elements S_3 [5]					
GS	-9962.23	-10357.49	-6225.60	$+1860.72$	$+12291.06$	-25.49789
KZ	-9971.92	-10359.59	-6223.36	$+1863.09$	$+12289.80$	-25.50357
GS-KZ	$+9.69$	$+2.10$	-2.24	-3.37	$+1.26$	$+0.00568$
	1964 Sept. 22.0 — 1967 Jan. 30.0 starting elements S_4 [6]					
GS	-3031.91	-2077.89	-1106.85	-1092.48	-1401.06	-4.74291
KZ	-3032.04	-2077.95	-1107.32	-1092.39	-1400.98	-4.74274
GS-KZ	$+0.13$	$+0.06$	$+0.47$	-0.09	-0.08	-0.00017

The results of the calculations are contained in the table 1. They are given in the form of perturbations of orbit osculation elements caused by the Jupiter and Saturn influence in several periods of the comet motion about the Sun. The perturbations have been calculated by the elements variation method (see e.g. [1]) by G. Sitarski (GS) and on the other hand by the numerical integration of the motion equations on the URAL-2 digital computer by the described program (KZ). The perturbations of the following orbit elements are given:

$$\begin{aligned}
\delta M &= M_k - M_p - n_p(t_k - t_p),\\
\delta\varphi &= \varphi_k - \varphi_p,\\
\delta\Omega &= \Omega_k - \Omega_p,\\
\delta\pi &= \pi_k - \pi_p,\\
\delta i &= i_k - i_p,\\
\delta n &= n_k - n_p,
\end{aligned}$$

where the indexes p and k denote the elements in the initial and final epoque respectively. The denotations of the starting elements systems are the same as in [4], [5], [6]. The integrations, the results of which are in the table 1, were performed with the constant step $h = 2^d$.

The respective differences (GS)-(KZ) show the good coincidence of the perturbation calculations results obtained by the two quite different methods. The comparatively great divergence in the period 1963 Sept. 8.0-1964 Sept. 22.0 one can justify by the relatively great values of the perturbations in this time interval. In this period the comet is in the neighbourghood of Jupiter and its smallest distance from this planet is 0.328041 astronomical unit [5]. The great influence of the proximity of Jupiter causes the respectively great perturbations in the comet motion. When calculating by an arithmometer it is convenient to perform calculations in the jovicentric system, considering the Sun and the other planets chiefly Saturn, as the bodies disturbing the hyperbolic comet motion with respect to Jupiter. In this way G. Sitarski has calculated the perturbations in the period 1963 Sept. 8.0-1964 Sept. 22.0 [5].

Table 2

1952 Mar. 9.0 — 1957 Jan 2.0						
	δM	$\delta\varphi$	$\delta\Omega$	$\delta\pi$	δi	δn
GS	+1218.28	+16.73	+22.00	−89.58	+28.74	+0.20255
$h = 2^d$	−0.49	−0.17	−0.06	−0.05	−0.04	−0.00062
$h = 4^d$	−2.93	−0.03	−0.06	−0.16	−0.04	−0.00216
$h = 8^d$	−54.13	+2.94	−0.05	−1.82	−0.02	−0.03456
$h = 16^d$	−1289.80	+72.90	+0.19	−18.43	+0.37	−0.82109

The increasing of the step of integration in the Runge-Kutta-Gill method causes the respectively greater differences between the perturbations obtained by this method and arbitrary constants variation method. The corresponding differences between the perturbations (GS) in the period 1952 Mar. 9.0 — 1957 Jan. 2.0 for the different steps of integration are contained in the table 2. Their rapid increase with the increasing step of integration shows the great dependence of integration errors on the value of the step of integration used in calculations.

§ 6. The results of computations accomplished on the URAL-2 electronic digital computer on the base of the program of integration of planetoid (comet) motion equation described in the paper, gathered in the tables 1 and 2, show:

a) The full correctness of the program.

b) The Runge-Kutta-Gill method of integration of the ordinary differential equations on digital computers is useful to the orbital problems of celestial mechanics.

c) Effectiveness of the used method of obtaining of the great planet coordinates from the tables prepared in advance especially for this aim.

The discussed program have been worked out by A. Naehr M. Sc. and the author of the paper under the supervision and guidance of Dr W. Turski. The author gives thanks to Dr G. Sitarski for many valuable remarks helpful in writing of this paper.

REFERENCES

[1] Dubjago A. D., 1949, *Opredelenie orbit*, GITTL, Moskva-Leningrad.
[2] Gill S., 1951, Proc. Cambridge Phils. Soc., 47, 96-108.
[3] Naehr A., 1964, *Computatio*, 2, 37-42.
[4] Sitarski G., 1964, Acta Astronomica, 14, 1-16.
[5] Sitarski G., 1964, Acta Astronomica, 14, 17-24.
[6] Sitarski G., 1964, Acta Astronomica, 14, 25-37.
[7] Ziołkowski K., 1965, *Computatio*, 3, 45-61.
[8] Ziołkowski K., 1965, *Computatio*, 3, 5-7.

ERRATA

Strona	Jest	Powinno być
66_7	points P_2 and P_{3_M}	points P_1 and P_{3_M}
	$B_\mu \wedge A_\mu \underset{1}{=} B_{\mu'} \wedge A'_\mu \wedge B_\mu$	$B_\mu \wedge A_\mu = B_{\mu'} \wedge A_{\mu'} \wedge B_\mu$
70^7	$\Sigma(P_\mu) \neq \Sigma(P'_\mu)$	$\Sigma(P_\mu) \neq \Sigma(P_{\mu'})$

Computatio nr 3

Państwowe Wydawnictwo Naukowe 1965

Wydanie I. Nakład 800+100 egz. Ark. wyd. 6,25; ark. druk. 6,5+1 wkl.
Papier druk. sat. kl. V 65 g, 70 × 100. Oddano do druku 6. XI. 1965 r.
Druk ukończono w maju 1966 r. Cena zł 18.—

Wrocławska Drukarnia Naukowa