COMPUTATION OF EPHEMERIDES FOR ARTIFICIAL EARTH SATELLITE ON THE ELECTRONIC DIGITAL COMPUTER URAL-2

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## I. Introduction

Within the cooperation between Academies of Sciences of socialistic countries in the field of optical observations of artificial earth satellites the contribution of the Computation Centre of the Polish Academy of Sciences is, among others, the problem of computing ephemerides for the American-Canadian ionospheric satellite Alouette (with the international notation $1962 \beta a-1$ ) for the use of satellite-tracking stations in the European people's republics and the Soviet Union.

The computations are for obtaining from given orbital elements a number of positions of the satellite with respect to the individual tracking station and the corresponding time moments. These quantities are satellite predictions a few days in advance, and in effect of certain factors disturbing satellite's motion only approximate position is determined. The observation of the satellite, i. e. the measurement of its actual position in the considered system of coordinates is then made much easier. Taking measurements from a number of tracking stations it is possible to make the appropriate corrections to orbital elements i.e. to up date them, what in turn may serve for various purposes. The most important of them are the following:

1) Actual orbital elements make possible to determine accurate positions of the setellite with respect to the Earth at the moments of measurements made by the satellite instruments (e.g. ionospheric, magnetic, meteorologic, etc.).
2) From a number of determinations of the orbital elements the changes in these elements can be obtained; analysis of the latter permits to deduce the causes of the changes, e.g. nonuniform distribution of masses of the Earth, its shape, and the density of upper atmosphere.

## II. Input data and the method assumptions

## 1. Orbital oloments of artificial satellite

The presented method for computing ephemerides is based on so called modified orbital elements with the use of principles given in [1]. The modified orbital elements describe a movement of the satellite in the system of coordinates that rotates with the Earth around its axis. The advantage of using the modified orbital elements for computing ephemerides is obvious from the fact that the coordinates of any point on the Earth (and consequently of a tracking-station) are constant in the considered system while in a motionless system they depend upon time.

A set of modified orbital elements is the following:

1) Epoch of perigee ( $\left.T_{0}\right)^{-}$- a moment expressed in the universal time (month, day, hours, minutes, and hundredth fractions of minutes) to which quantities of the remaining elements are referred. In this moment the satellite is at the perigee.
2) Orbit inclination ( $i$ ) with respect to the equator in degrees and the hundredth fractions of degrees.
3) Western longitude of the ascending node ( $L_{0}$ ) - an angle measured in the plane of the equator from the Greenwich meridian to the ascending node towards West (in degrees and the hundredth fractions of degrees).
4) Mean planar day (MPD) $\left(1^{d}+D\right)$ - time interval between two successive passes of the orbit plane throngh the same point on the Earth. This quantity includes the daily rotation of the Earth and the absolute motion of the node. The MPD is mostly to be given in the form of $1^{d}+D$, where $D$ is in minutes and hundredth fractions of minutes.
5) Argument of perigee ( $\omega_{0}$ ) - an angle measured in the plane of the orbit from the ascending node to the perigee according to the direction of the satellite movement (in degrees and hundredth fractions of degrees).
6) Change in the argument of perigee per one period ( $\Delta \omega$ ) in thousandth fractions of degrees.
7) Anomalistic period $\left(P_{0}\right)$ - time interval between two successive passes of the satellite through the perigee; this quantity includes the perigee motion (point 6); it is to be given in minutes and thousandth fractions of minutes.
8) Change in anomalistic period per one period ( $\Delta P$ ) - in hundred thousandth fractions of a minute.
9) Eccentricity of the orbit (e) - five digits after the point.
10) Geocentric distance of the perigee ( $r_{p}$ ) - in miles ( 1 mile $=1 \cdot 609 \mathrm{~km}$ ).

Besides a set of the orbital elements in the above form the following data are to be introduced:
11) Estimated correction to crossing moments ( $\tau$ ) in minutes; e.g. $\tau=+4$ means that the satellite will appear 4 min. later than it results from the computations disregarding this correction. It is obtained from the satellite's observations already made.
12) Beginning of the time interval for which the ephemerides are to be computed ( $T_{\text {begin }}$ ) - month and day of a month.
13) End of the time interval for which the ephemerides are to be computed ( $T_{\text {end }}$ ) - moth and day of a month.

## 2. Data on satellite-tracking stations

In order to compute ephemerides for an artificial earth satellite it is required to know the geographic coordinates of the stations for which the ephemerides are to be computed. The programme based on the presented method permits to compute simultaneously the ephemerides for no more than 30 stations within the geographic region of 0.5 in longitude and $5^{\circ}$ in latitude. In the ephemerides for stations lying in a region larger than that rather great errors are found. Geographic longitudes (Eastern) of stations are to be given in hours and decimal fractions of hours, while latitudes (Northern) - in degrees and decimal fractions of degrees.

More detailed description of the method for preparing the input data and feeding them into the computer is given in section V .

## 3. Form of an ephemeris

The ephemeris to be computed for the satellite at a given point on its track is to be considered as a set of three quantities:

1) time ( $T$ ),
2) azimuth measured clockwise from North (A),
3) horizontal altitude ( $h$ ) - vertical angle measured from the horizon.

For every flight of the satellite and for every station from which it can be observed the ephemerides are to be given for two points on satellite's track, i. e.:

1) that point at which the satellite is nearest to the station, i. e. the point on satellite's track with the greatest horizontal altitude (this point is further denoted by $Z$ );
2) that point at which the satellite crosses the station's meridian (denoted by $M$; the same denotation for mean anomaly does not lead further to ambiguity), i. e. the point on satellite's track with azimuth $A_{M}$ equal to $0^{\circ}$ or $180^{\circ}$. If, however, the satellite which is visible at point $Z$ becomes invisible or hard to be observed at point $M$, then we have to compute and give the ephemeris for such point on the track (denoted by $X$ )
which is to be crossed by the satellite 2 min . earlier or later than point Z.

The observation of the satellite in the expected time and place according to the ephemeris is possible if the following conditions for visibility and configuration of points $Z$ and $M$ or $X$ will be satisfied:

1) Time moments for which the ephemerides are given should refer to the time after the nautical evening twilight ( $\mathrm{n} . \mathrm{e} . \mathrm{t}$. ) and before the nautical morning twilight (n. m.t.) at a given station making optical observations. (As the n.e.t. and the n.m.t. we assume time moments at which the Sun is $12^{\circ}$ below the horizon after its set and before its rise).
2) At these moments the satellite should be illuminated by the Sun, i. e. it should be out of the Earth's shadow.
3) Since the effect of atmospheric refraction rapidly increases near the horizon, then horizontal altitude $h$ of the satellite at point $Z$ should be greater than $22^{\circ}$ while at point $M$ or $X$ greater than $15^{\circ}$.
4) In order that the satellite be observed at both point $Z$ and $M$ they must be separated by minimum 2 min . in time.

In the case when any of conditions 1,2 or 3 is not satisfied for point $Z$, then no ephemeris for a definite flight and a definite station is to be given. If any of conditions $1,2,3$ or 4 is not satisfied for point $M$, then the ephemeris is to be computed for point $X$. In the case when the above conditions are satisfied for point $Z$, then they must also be satisfied al least for one variant of point $X$ (i. e. a point lying before or behind point $Z$ ).

## 4. Required accuracy of ephemeris

Accuracy of the computed ephemeris depends on both the accuracy of the orbital elements at a given epoch and their stability. Because of disturbances effecting from the atmospheric drag, oblateness of the Earth and non-homogenous distribution of its masses, etc., the orbital elements are subjected to quite considerable changes in time. For that reason they can be extrapolated (as it is usually accomplished for ephemeris purposes) only for relatively short interval of time starting with the perigee epoch $T_{0}$. The longer this interval the greater errors in ephemerides. (If, for example the interval for which the orbital elements are to be extrapolated is 4 weeks, then the computed crossing times for satellite Alouette will have the error of the order of $\pm 2 \mathrm{~min}$.).

The required accuracy of ephemerides for purposes of optical observations (visual and photographic) follows from a magnitude of the field of vision in the appropriate instruments; the satellite should be possibly near the centre of this field when directing the telescope according to the ephemeris. It is usually assumed that the accuracy of horizontal coordi-
nates should be of the order of $1^{\circ}$ and the accuracy of time of the order of 1 min . And this is the accuracy with which the ephemerides are given to individual stations.

## III. Numerical method

## 1. General principles

1) Within a geographic region containing the stations a so called reference point is to be chosen with longitude being arithmetic mean of longitudes of all stations; on the other hand, the reference latitude is either the least of the latitudes of all stations (in the case when orbit inclination is greater than $\varphi \mathrm{min}$ ) or the reference latitude is assumed to be equal to the orbit inclination (if the latter is less or equal to $\varphi \mathrm{min}$ ).

$$
\begin{gathered}
\lambda_{0}=\lambda_{\text {maan }}=\frac{\lambda_{1}+\lambda_{2}+\ldots+\lambda_{n}}{n} \\
\varphi_{0}=\varphi_{\text {min }} \quad \text { if } \quad i>\varphi_{\text {min }} \\
\varphi_{0}=i \quad \text { if } \quad i \leqslant \varphi_{\text {min }}
\end{gathered}
$$

2) The discussed method and the programme based on it are applicable only to the positive orbit inclinations and to the stations in the Northern hemisphere. Thus the above principle in determining the reference latitude renders impossible to mark this latitude as negative.
3) The quantities referring to the ascending branch (i.e. to this part of the orbit in which the satellite crossing the equator passes from the southern hemisphere into the northern one) are denoted by the upper index $A$. On the other hand, the quantities referring to the descending branch are denoted by index $D$.

## 2. Determination and choice of times when the reference point is in the orbit plane

One of the factors disturbing satellite movement is rotation of the orbit plane round the Earth's axis westwards with the speed of a few degrees per day. This rotation, independent of the 24 hrs . movement of the Earth is distinguished by a continuous change in direction of line of nodes in the motionless system of coordinates (i. e. connected with the vernal equinox).

On the other hand, the Earth rotates eastwards with the period of 24 hrs . Therefore, for an observer on the Earth's surface a period of the apparent rotation of the orbit plane is shorter by a few minutes than the period of the Earth rotation and it is named mean planar day (MPD).

In one such period all points on the Earth's surface within the latitudes $-i<\varphi<+i$ are to be found in the orbit plane twice, once in its ascending part, and for the second time in its descending part.

The quantity $l$, i. e. a difference in longitude between the ascending node and the reference point at


Fig. 1. Orbit's projection on Earth's surface at moment of its transit through the reference point 0 the moment at which the latter crosses the orbit plane can be determined by solving the appropriate rectangular spherical triangle (Fig. 1).

$$
\begin{equation*}
\sin l=\frac{\operatorname{tg} \varphi_{0}}{\operatorname{tg} i} \tag{1}
\end{equation*}
$$

Hence we obtain two solutions, $l^{A}$ and $l^{D}$ for the ascending branch and the descending one, respectively. In the case when $\varphi_{0}=i$ we have one solution $l=6^{h}$ (all longitudes are to he expressed in hours because of their relation to time). The quantities $l$ permit to determine Western longitudes of the ascending node $L$ at the moments at which the reference point 0 crosses the orbit plane.

$$
\left.\begin{align*}
& L^{A}=24^{h}-\lambda_{0}+l^{A}  \tag{2a}\\
& L^{D}=24^{h}-\lambda_{0}+l^{D}
\end{align*} \right\rvert\, \quad \text { if } \quad \varphi_{0}=\varphi_{\mathrm{mln}}
$$

or

$$
\begin{equation*}
L=24^{h}-\lambda_{0}+l \quad \text { if } \quad \varphi_{0}=i \tag{2b}
\end{equation*}
$$

Thus the time interval needed for the orbit plane to rotate from position $L_{0}$ (at $T_{0}$ ) to $L$ (running the path $L-L_{0}$ ) will be:

$$
\begin{equation*}
\Delta T^{A}=\frac{L^{A}-L_{0}}{\mu} \quad \text { and } \quad \Delta T^{D}=\frac{L^{D}-L_{0}}{\mu} \tag{3a}
\end{equation*}
$$

if

$$
\varphi_{0}=\varphi_{\min }
$$

or

$$
\begin{equation*}
\Delta T=\frac{L-L_{0}}{\mu} \quad \text { if } \quad \varphi_{0}=i \tag{3b}
\end{equation*}
$$

where $\mu$ is a velocity of the apparent rotation of the orbit plane,

$$
\begin{equation*}
\mu=\frac{24^{\Lambda}}{24^{\frac{1}{2}}+D}=\frac{24^{\Lambda}}{D^{\prime}}, \tag{4}
\end{equation*}
$$

and $D^{\prime}=24^{h}+D$ is a mean planar day.
Therefore we can write

$$
\begin{equation*}
\Delta T=\frac{D^{\prime}}{24^{2}}\left(24-\lambda_{0}+l-L_{0}\right) \tag{5}
\end{equation*}
$$

where $\Delta T$ is expressed in days.
Now the successive moments at which the reference point crosses the orbit plane can be determined:

$$
\begin{array}{ll}
T_{1}^{A}=T_{0}+\Delta T^{A} & T_{1}^{D}=T_{0}+\Delta T^{D} \\
T_{2}^{A}=T_{1}^{A}+D^{\prime} & T_{2}^{D}=T_{1}^{D}+D \\
T_{3}^{A}=T_{2}^{A}+D^{\prime} & T_{2}^{D}=T_{2}^{D}+D^{\prime}  \tag{6}\\
\cdots \cdots \cdots & \cdots \cdots \cdots \\
T_{i+1}^{A}=T_{i}^{A}+D^{\prime} & T_{i+1}^{D}=T_{i}^{D}+D^{\prime}
\end{array}
$$

Since $D^{\prime}$ is nearly $1440^{m}=1^{d}$ then, if $\varphi_{0}=\varphi_{\min }$ for a given 24 hrs period we have usually only two moments in which the reference point 0 is in the orbit plane, namely $T_{i}^{A}$ and $T_{i}^{D}$. Flights of the satellite in the time intervals $\left[\left(T_{i}^{A}-\frac{1}{2} P\right),\left(T_{i}^{A}+\frac{1}{2} P\right)\right]$ and $\left[\left(T_{i}^{D}-\frac{1}{2} P\right),\left(T_{i}^{D}+\frac{1}{2} P\right)\right]$ are then the flights nearest to point 0 within a given 24 hrs period.

However, the satellite can be visible not only during these nearest flights but also during the neighbouring flights, i.e. the ones ocurring one period earlier and later. Altogether we will have then the following set of moments defining separate flights
(7)
$\begin{cases}T_{11}^{A}=T_{10}^{A}-P & T_{11}^{D}=T_{10}^{D}-P \\ T_{10}^{A} & T_{10}^{D} \\ T_{12}^{A}=T_{10}^{A}+P & T_{12}^{D}=T_{10}^{D}+P \\ T_{21}^{A}=T_{20}^{A}-P & T_{21}^{D}=T_{20}^{D}-P \\ T_{20}^{A} & T_{20}^{D} \\ T_{22}^{A}=T_{20}^{A}+P & T_{22}^{D}=T_{20}^{D}+P \\ \cdots \cdots \cdots & \cdots \cdots \cdots \\ \cdots \cdots \cdots & \cdots \cdots \cdots \\ \cdots \cdots \cdots & \cdots \cdots \cdots \cdots \\ T_{i 1}^{A}=T_{i 0}^{A}-P & T_{i 1}^{D}=T_{i 0}^{D}-P \\ T_{i 0}^{A} & T_{10}^{D} \\ T_{i 2}^{A}=T_{i 0}^{A}+P & T_{i 2}^{D}=T_{i 0}^{D}+P .\end{cases}$

It is obvious that only in the moments with the second index equal zero, i. e. in the ones computed according to (5) and (6) the reference point is in the plane of the orbit.

Two values for $T_{i j}$ are obtained if $\varphi_{0}<i$. If, however,

$$
\varphi_{0}=i \quad \text { then } \quad T_{i j}^{A}=T_{i j}^{D}=T_{i j} .
$$

The fact that the satellite must appear within the limits of $\pm \frac{1}{2} P$ from $T_{i j}$ makes possible to eliminate the moments between n.m.t. and n.e.t. within the region of stations. Moreover, we can eliminate from further computations the moments falling between sunset and sunrise at satellity's height above the reference point; during the flights defined by these moments the satellite is in the Earth's shadow above the stations region and therefore is invisible (provided that it is not supplied with own light source).

In consideration of the above remarks the conditions to be satisfied by moments $T_{i j}$ are as follows:

$$
\begin{equation*}
T_{i j}-\frac{1}{2} P \leqslant T_{\text {n.m.t. }}^{\max } \quad \text { and } \quad T_{i j}+\frac{1}{2} P \geqslant T_{\text {sunrlso }} \tag{8a}
\end{equation*}
$$

or

$$
\begin{equation*}
T_{i j}-\frac{1}{2} P \leqslant T_{\text {sunset }} \quad \text { and } \quad T_{i j}+\frac{1}{2} P \geqslant T_{\text {n.e.t. }}^{\min } \tag{8b}
\end{equation*}
$$

$T_{\text {n.m.t. }}^{\max }$ and $T_{\text {n.e.t. }}^{\min }$ mean here the latest moment of the nautical morning twilight and the earliest moment of the nautical evening twilight within the region of stations.

The moments $T_{i j}$ non-satisfying the conditions (8a) and (8b) are rejected since during the flights defined by them the satellite cannot be observed from any station within a given geographical region. On the other hand, during the flights defined by non-eliminated moments $T_{i j}$ the satellite can (but not must) be seen within the region of stations.

Moments $T_{\text {n.m.t. }}^{\operatorname{man}}$ and $T_{\mathrm{n} ., \mathrm{e}, \mathrm{L}}^{\mathrm{mln}}$ are to be computed according to the following approximate formulae neglecting the equation of time which is here a negligible quantity.

$$
\left\{\begin{array}{l}
T_{\text {n.m..t. }}^{\operatorname{man}} \cong t_{\text {n.m.t. }}-\lambda^{\min }-12^{A}  \tag{9}\\
T_{\text {ne.e.t. }}^{\min } \cong t_{\text {n.e.t.t. }}-\lambda^{\max }-12^{\Lambda}
\end{array}\right.
$$

where $\lambda^{m / n}$ and $\lambda^{\text {max }}$ are the longitudes of the stations situated respectively farthest eastwards and farthest westwards in a given region;
$t_{\text {n.m.t. }}, t_{\text {n.c.t. }}$ are hour angles of the Sun at the moments of $\mathrm{n} . \mathrm{m} . \mathrm{t}$. and n.e.t.; these angles are given by

$$
\begin{equation*}
\cos t=\frac{\sin h^{\circ}-\sin \varphi_{\operatorname{mesn}} \sin \delta^{\circ}}{\cos \varphi_{\operatorname{mean}} \cos \delta^{\circ}} \tag{10}
\end{equation*}
$$

while

$$
\begin{aligned}
& 12^{h}<t_{\text {n.m.t. }}<24^{h} \\
& 0^{h}<t_{\text {n.e.t. }}<12^{h}
\end{aligned}
$$

$\varphi_{\text {mas }}$ - mean latitude of stations in a given region,
$h^{\circ}$ - horizontal altitude of the Sun; at the moments of n.m.t. and n. e. t. $h^{\circ}=-12^{\circ}$ is to be taken;
$\delta^{\circ}$ - mean declination of the Sun for the time interval for which the ephemerides of the satellite are to be computed.
The formulae (9) and (10) are used also for computing the moments $T_{\text {sunrise }}$ and $T_{\text {sunset }}$, with substitution of the coordinates $\lambda_{0}$ and $\varphi_{0}$ of the reference point instead of $\lambda^{\min }, \lambda^{\max }$ and $\varphi_{\text {mean }}$. As $h^{\circ}$ we substitute here an angle of lowering the horizon $(-x)$ computed from

$$
\cos x=\frac{R}{r_{a}}
$$

where $R$ is the Earth's radius and $r_{a}$ - the radius-vector of the apogee.

## 3. Determination of the last moment in which satellite is at the perigee before given $T_{i j}$

For determining the last moment at which satellite is at the perigee a number of revolutions from $T_{0}$ to given $T_{i j}$ should be known. First approximation of this number is to be found by dividing the length of time interval $\left(T_{i j}-T_{0}\right)$ by the anomalistic period $P_{0}$ :

$$
\begin{equation*}
N^{\prime}=\frac{T_{i}-T_{0}}{P_{0}} \tag{11}
\end{equation*}
$$

Now, using $N^{\prime}$ we can determine the current anomalistic period $P$ and the mean period $P_{\text {ream }}$ in the interval ( $T_{i j}-T_{0}$ ).

$$
\begin{gather*}
P=P_{0}+N^{\prime} \cdot \Delta P  \tag{12}\\
P_{\text {moan }}=\frac{1}{2} P_{0}+P=P_{0}+\frac{1}{2} N^{\prime} \cdot \Delta P \tag{13}
\end{gather*}
$$

where $\Delta P$ is a given change in the anomalistic period per one revolution.
The period $P_{\text {mean }}$ serves in turn for determining the exact number of revolutions from the epoch of perigee

$$
\begin{equation*}
N=\frac{T_{i j}-T_{0}}{P_{\text {mean }}} \tag{14}
\end{equation*}
$$

Since in the epoch of perigee the satellite was at the perigee, then after $\bar{N}$ cycles (where $\bar{N}$ is the integer) it is also to be found at the perigee. Hence the sought last moment at which the satellite crosses the perigee is:

$$
\begin{equation*}
T_{p}=T_{0}+\bar{N} \cdot P_{\text {moan }} \tag{15}
\end{equation*}
$$

The above formula is to be regarded as the approximate one since the current period $P$ is not a linear function of time and its change per one revolution is not a constant. Therefore, the method suffers here from a loss in accuracy, and so the lower the satellite orbit and greater its eccentricity the greater loss in accuracy.

## 4. Moment at which satellite crosses the reference latitude and the longitude of crossing point

If we know a number of revolutions the current position of the perigee can be determined (argument of perigee)

$$
\begin{equation*}
\omega=\omega_{0}+N \cdot \Delta \omega \tag{16}
\end{equation*}
$$

where $\Delta \omega$ is a given change in the perigee position per one revolution.

The change in the argument of perigee per one revolution $\Delta \omega$ is, like the change in period $\Delta P$, not a constant in time. The approximate character of formula (16) is then also to be regarded as a factor lowering accuracy of the method.

The angular distance from the perigee to the point at which the satellite crosses the reference latitude $\varphi_{0}$ (point $S$ ) measured in the plane of the orbit (true anomaly) is:

$$
\begin{equation*}
v_{S}=\psi-\omega \tag{17}
\end{equation*}
$$

where $\psi$ is a central angle between the equator and the reference latitude measured in the plane of the orbit. This angle is to be found from the rectangular spherical triangle $\mathscr{A} 00^{\prime \prime}$ (Fig. 2):

$$
\begin{equation*}
\sin \psi=\frac{\sin \varphi_{0}}{\sin i} . \tag{18}
\end{equation*}
$$

In the case when $\varphi_{0}<i$ two values $\psi^{4}$ and $\psi^{D}$ are obtained for $\psi$. If, however, $\varphi_{0}=i$ then $\psi^{A}=\psi^{D}=\psi$. For computing $v_{S}$ according to (17) we should take the value of $\psi$ corresponding to the same branch as moment $T_{i j}$.

Using true anomaly $v$ computed as above we determine mean anomaly $M$ by means of the eccentric anomaly $E$. The latter is defined by the formula:

$$
\begin{equation*}
\operatorname{tg} \frac{E}{2}=\frac{\operatorname{tg} \frac{v}{2}}{\sqrt{\frac{1+e}{1-e}}} \tag{19}
\end{equation*}
$$

where $e$ is the orbit eccentricity. This anomaly is related with the mean anomaly by the Kepler's equation:

$$
\begin{equation*}
M=E-e \cdot \sin E \tag{20}
\end{equation*}
$$

In our computations we use so called periodical equivalent of the true anomaly $v_{S}$. This is the time interval expressed as a part of the period required for the satellite to run the distance $v_{S}$. It is to be obtained by dividing $M$ by $2 \pi$, as the mean anomaly determines the position of a fictitious satellite which moves along the orbit with the constant velocity equal to the mean velocity of the true satellite. Let $\bar{M}$ denote the periodical equivalent. Then we have:

$$
\begin{equation*}
M=\frac{M}{2 \pi} \tag{21}
\end{equation*}
$$

and the mentioned time interval required for the satellite to run the distance $v_{S}$, i. e. to pass from the current perigee to the reference latitude $\varphi_{0}$, will be in time units:

$$
\begin{equation*}
\delta T_{S}=\bar{M}_{S^{*}} P \tag{22}
\end{equation*}
$$

Hence the moment at which the satellite crosses the reference latitude is:

$$
\begin{equation*}
T_{S}=T_{p}+\delta T_{S}+\tau \tag{23}
\end{equation*}
$$

where $\tau$ is an estimated correction to crossing times obtained from the observations already made.

Moment $T_{S}$ refers to a given flight of the satellite defined by $T_{i f}$. Taking into account that at moment $T_{i 0}$ the reference point 0 is in the plane of the orbit the difference of longitudes between the points 0 and $S$ can be readily determined. This quantity expressed in hours will be:

$$
\begin{equation*}
l_{S}=24 \cdot\left(T_{S}-T_{i 0}\right) \cdot \mu \tag{24}
\end{equation*}
$$

where $\mu$ is computed according to (4) while $T_{S}$ and $T_{i j}$ are expressed in days. The positive value of $l_{S}$ means that the satellite crosses the latitude
$\varphi_{0}$ (point $\mathbb{S}$ ) westwards from meridian $\lambda_{0}$. The absolute Eastern longitude of point $\mathbb{S}$ will be then:

$$
\begin{equation*}
\lambda_{S}=\lambda_{0}-l_{S} \tag{25}
\end{equation*}
$$

## 5. Determination of the apparent direction of the satellite's flight

Because of the Earth's rotation the true direction of the satellite fllight defined by angle $q$ (Fig. 2) does not coincide with the direction of the flight to be seen by the observer at the point 0 on the Earth. If in the determined time moment the satellite crosses latitude $\varphi_{0}$ (i.e.


Fig. 2. Real and apparent direction of satellite's motion
point 0 coincides with point $S$ ) then after the elapse of infiintely small time interval $d t$ point 0 will shift to the position 0 , and

$$
00^{\prime}=\frac{2 \pi}{l^{d}} \cos \varphi_{0} d t ;
$$

the satellite will occupy then a new position $S^{\prime}$ distant from $S$ by the quantity

$$
S S^{\prime}=\frac{2 \pi}{P} d t
$$

The quantity $2 \pi / P$ is the mean angular velocity of the satellite and differs from its current angular velocity by quantity depending on orbit eccentricity and current satellite's position. Because of that the undermentioned formulae are approximate ones and valid only for small eccentricities (e. i. not exceeding 0.2).

Since the quantities $00^{\prime}$ and $S S^{\prime}$ are small, then considering the problem in the plane we have the following components of the shift $\mathbf{S S}^{\prime}$ :

$$
\delta S^{\prime} \varphi=\frac{2 \pi}{P} \sin q d t
$$

and

$$
S S_{\lambda}^{\prime}=\frac{2 \pi}{P} \cos q d t
$$

The sought apparent direction of the satellite flight defined by the angle $\Delta^{\Delta}$ measured from North is then:

$$
\tan \Delta^{A}=\frac{S S^{\prime} \varphi-00}{S S^{\prime} \lambda}
$$

Expressing the anomalistic period in minutes, and after dividing both the numerator and denominator by $d t$ we obtain

$$
\begin{equation*}
\tan \Delta^{A}=\frac{\sin q-\frac{P}{1440} \cos \varphi}{\cos q} \tag{26}
\end{equation*}
$$

The angle $q$ is to be detcrmined from the rectangular spherical triangle $\Omega 00^{\prime \prime}$ (fig. 2):

$$
\begin{equation*}
\sin q=\frac{\cos i}{\cos \varphi_{0}} \tag{27}
\end{equation*}
$$

Formula (26) is applied, of course, only to the case when we deal with the positive orbit inclination.

In the case $i<0$ we would have

$$
\tan \Delta^{4}=\frac{\sin q+\frac{P}{1440} \cos \varphi_{0}}{\cos q}
$$

The above considerations are valid only for the ascending branch while the angle $\Delta^{\Delta}$ obtained from (26) is in the Ist or IVth quadrant. For the descending branch we obtain

$$
\begin{equation*}
\Delta^{D}=\pi-\Delta^{L} . \tag{28}
\end{equation*}
$$

If, however, $\varphi_{0}=i$ we have $\Delta^{\Delta}=\Delta^{D}=\pi / 2$.

The quantities $T_{s}, l_{s}$ and $\Delta$ defining the time moment, the direction in which the satellite crosses $\varphi_{0}$, and the longitude of the crossing point permit to determine time moments and positions of the satellite with respect to given station at chosen points on the track. The exemplary


Fig. 3
configuration of the satellite's track (in projection on the Earth's surface) and the station is illustrated in Fig. 3 with the following denotations:

B - Earth's pole,
0 - Reference point,
I - Tracking station,
$S$ - Point at which the satellite crosses $\varphi_{0}$,
$Z$ - Point of the closest approach of the satellite to station $I$,
$M$ - Point at which the satellite crosses the station's meridian.
We still have

$$
\begin{array}{rlrl}
l_{S} & =\Varangle O B S & m & =\breve{S I} \\
l & =\Varangle S B I & w_{Z} & =\breve{S Z} \\
a & =\Varangle B S I & w_{M} & =\breve{S M} \\
\beta & =\Varangle S I B & d_{Z} & =\breve{I Z} \\
\delta_{Z} & =\Varangle S I Z & d_{M} & =\breve{I M} .
\end{array}
$$

From Fig. 3 we see that:

$$
\begin{equation*}
l_{I}=\lambda_{l}-\lambda_{0}+l_{S} \tag{29}
\end{equation*}
$$

On the other hand, from the triangle $S B I$ we can determine $\alpha, \beta$, and $m$ :

$$
\tan \frac{\alpha+\beta}{2}=\frac{\cos \frac{1}{2}\left(\varphi_{0}-\varphi\right)}{\sin \frac{1}{2}\left(\varphi_{0}+\varphi\right)} \cot \frac{l_{I}}{2}
$$

(30)

$$
\begin{align*}
\tan \frac{\alpha-\beta}{2} & =\frac{\sin \frac{1}{2}\left(\varphi_{0}-\varphi\right)}{\cos \frac{1}{2}\left(\varphi_{0}-\varphi\right)} \cot \frac{l_{I}}{2} \\
\sin m & =\frac{\sin l_{l} \cos \varphi_{0}}{\sin \beta} \tag{31}
\end{align*}
$$

Introducing the auxiliary denotations:

$$
d_{M}^{\prime}=\overline{B M}
$$

and

$$
\Delta^{\prime}=\Delta \quad \text { if } \quad l^{\prime}>0
$$

or

$$
\Delta^{\prime}=\pi-\Delta \quad \text { if } \quad l^{\prime}<0
$$

from the triangle $S B M$ we have:

$$
\left\{\begin{array}{l}
\tan \frac{1}{2}\left(d_{M}^{\prime}+w_{M}\right)=\tan \frac{1}{2}\left(\frac{\pi}{2}-\varphi_{0}\right) \frac{\cos \frac{1}{2}\left(\Delta^{\prime}-l_{I}\right)}{\cos \frac{1}{2}\left(\Delta^{\prime}+\left|l_{I}\right|\right)}  \tag{32}\\
\tan \frac{1}{2}\left(d_{M}^{\prime}-v_{M}\right)=\tan \frac{1}{2}\left(\frac{\pi}{2}-\varphi_{0}\right) \frac{\sin \frac{1}{2}\left(\Delta^{\prime}-l_{I}\right)}{\sin \frac{1}{2}\left(\Delta^{\prime}+l_{I}\right)}
\end{array}\right.
$$

thus $d_{M}^{\prime}$ and $w_{M}$ will be determined.
Further, by denoting

$$
\gamma=\Delta^{\prime}-a
$$

we shall compute $d_{M}$ as:

$$
\left\{\begin{array}{lll}
d_{M}=d_{M}^{\prime}-\frac{\pi}{2}+\varphi & \text { if } & \gamma>0  \tag{33}\\
d_{M}=\frac{\pi}{2}-\varphi-d_{M}^{\prime} & \text { if } & \gamma<0 .
\end{array}\right.
$$

The azimuth of point $M$ will be:

$$
\left\{\begin{array}{lll}
A_{M}=\pi & \text { if } & 0<\gamma<\pi  \tag{34}\\
A_{M}=0 & \text { if } & \pi<\gamma<2 \pi .
\end{array}\right.
$$

From the rectangular triangle $S I Z$ we shall determine $d_{Z}, w_{Z}$, and $\delta_{Z}$ :

$$
\begin{align*}
& \sin d_{Z}=\sin m \sin \gamma  \tag{35}\\
& \tan w_{Z}=\tan m \cos \gamma  \tag{36}\\
& \cot \delta_{Z}=\cos m \tan \gamma . \tag{37}
\end{align*}
$$

Using $\delta_{z}$ and $\beta$ the azimuth of point $Z$ can be directly determined:

$$
\left\{\begin{array}{lll}
A_{Z}=2 \pi-\left(\beta+\delta_{Z}\right) & \text { if } & l_{I}>0  \tag{38}\\
A_{Z}=\beta+\delta_{Z} & \text { if } & l_{I}<0 .
\end{array}\right.
$$

In the case $l_{I}<0$ the signs of $w_{M}$ and $w_{Z}$ should be reversed.
If the conditions for visibility of the satellite at the point $M$ are not satisfied (with satisfied conditions at the point $Z$ ) then the other point for which the ephemeris is to be given is point $X$, distant from $Z$ by $\Delta w=+7^{\circ}$ or by $\Delta w=-7^{\circ}$. The quantity $\pm 7^{\circ}$ effects from the assumption that the satellite is to cross point $X$ about 2 min . earlier or later than point $Z$; the satellite with the anomalistic period of about 105 min . and with the orbit nearly circular (as in the case of Alouette) has the geometric angular velocity almost constant and equal to about $3^{\circ} .5 / \mathrm{min}$.

From the triangle $I X Z$ the quantity $\Delta \delta$ can be determined:

$$
\begin{equation*}
\tan \Delta \delta=\frac{\tan \Delta_{w}}{\sin d_{z}} \tag{39}
\end{equation*}
$$

hence

$$
\begin{equation*}
\delta_{X}=\delta_{Z}+\Delta \delta \tag{40}
\end{equation*}
$$

If $v_{Z}$ and $\delta_{Z}$ have the same signs then $\Delta \delta$ is positive; if, however, they have different signs $\Delta \delta$ is negative.

The quantity $d_{S}$ is to be determined by solving the triangle $I X S$ :

$$
\begin{equation*}
\sin d_{X}=\frac{\sin w_{X} \sin \gamma}{\sin \delta_{X}} \tag{41}
\end{equation*}
$$

The azimuth of point $X$ can be determined in the similar way as the azimuth of point $Z$, using (38):

$$
\begin{array}{lll}
A_{X}=2 \pi-\left(\beta+\delta_{X}\right) & \text { if } & l_{I}>0  \tag{42}\\
A_{X}=\beta+\delta_{X} & \text { if } & l_{I}<0 .
\end{array}
$$

The quantities $d$ and $w$ referring to the points $Z, M$ and $X$ make possible to determine horizontal altitudes $h$, and the geocentric and topocentric distances to these points, as the true anomaly of each of them will be:

$$
\begin{equation*}
v=v_{S}+v \tag{43}
\end{equation*}
$$

and the geocentric distance:

$$
\begin{equation*}
r=\frac{1+e}{1+e \cos v} r_{p} \tag{44}
\end{equation*}
$$

where:
$v_{S}$ - is the true anomaly of point $S$ computed according to (17),
$e$ - eccentricity of the orbit,
$r_{p}$ - geocentric distance to the perigee.
Now, from Fig. 4 we shall determine the topocentric distance $\varrho$ and the horizontal altitude $h$ :

$$
\begin{gather*}
\varrho^{2}=R^{2}+r^{2}-2 R r \cos d  \tag{45}\\
\cos h=\frac{r \sin d}{\varrho}
\end{gather*}
$$

where $R$ is a mean radius of the Earth; $R=6370 \mathrm{~km}$. For removing ambiguity of $\cos h$ we have the following relation: if $r \cos d<R$ then $h<0$.

For determining the moments at which the satellite is to cross the points $Z, M$ and $X$ we shall com-


Fig. 4 pute the corresponding periodic equivalents $\bar{M}$ using (19), (20) and (21); for the true anomaly we shall substitute the appropriate values computed according to (43). The time intervals required for the satellite to pass from the reference latitude (point $S$ ) to the points $Z, M$ and $X$ will be the following:

$$
\begin{equation*}
\delta T=\bar{M} \cdot P-\delta T_{s} \tag{47}
\end{equation*}
$$

where $P$ is the current anomalistic period computed according to (12) and $\delta T_{S}$ is computed according to (22).

Therefore the corresponding moments at which the satellite is to cross the points $Z, M$ and $X$ will be:

$$
\begin{equation*}
T=T_{S}+\delta T=T_{S}+\bar{M} \cdot P-\delta T_{S} \tag{48}
\end{equation*}
$$

## 7. Checking the conditions for satellite's illumination and visibility

In section II, item 3 we give the conditions to be satisfied by the satellite's positions $Z, M$ and $X$, and the corresponding time moments. Checking the conditions 3 and 4 is performed directly with the use of the computed horizontal altitudes $h_{Z}, h_{M}$ and $h_{X}$ and the moments of crossings $T_{Z}$ and $T_{M}$. However, for checking the conditions 1 and 2 some auxiliary computations are to be performed.

The condition 1 can be written as follows:

$$
\begin{equation*}
T_{Z}<T_{\text {n.m.t. }} \quad \text { or } \quad T_{Z}>T_{\text {n.e.t. }} \tag{49}
\end{equation*}
$$

what means that the moment at which the satellite is to cross point $Z$ should be at night for a given station. The moments of n.m.t. and n.e.t. are to be computed from the following formulae:

$$
\left\{\begin{array}{l}
T_{\text {n.m.t. }}=t_{\text {n.m.t. }}-\eta-\lambda  \tag{50}\\
T_{\text {n.e.t. }}=t_{\text {n.e.t. }}-\eta-\lambda
\end{array}\right.
$$

where $t_{\text {n-m_t. }}, t_{\text {nee.t. }}$ are the hour angles of the Sun at the moments of n. m.t. and n.e.t. computed according to (10):
$\eta$ - equation of time for a given date,
$\lambda$ - longitude of a given station.
Using (10) we make the following substitutions:
for $\varphi_{\text {mann }}$ - latitude of a given station $(\varphi)$,
for $\delta^{\circ}$ - current declination of the Sun,
$h^{\circ}=-12^{\circ}$.
For checking condition 2 (illumination of the satellite by the Sun) we have to determine the moments of sunset and sunrise at the points $Z$ and $M$ or $X$ (on the height of the satellite flight). The corresponding crossing times should be then such that

$$
\begin{equation*}
T_{\text {sunfise }}<T<T_{\text {sunset }} . \tag{51}
\end{equation*}
$$

The moments $T_{\text {sunrise }}$ and $T_{\text {sunset }}$ may also be determined from (50), however, we must compute here the geographic coordinates of projections on the Earth's surface of the points $Z$ and $M$ or $X$, and the angle of lowering the horizon. For $\delta^{\circ}$ and $\eta$ their current values are to be substituted.

The coordinates of the satellite at the points $Z$ and $M$ or $X$ are to be determined by solving the spherical triangle in Fig. 5. We have

$$
\begin{equation*}
\sin \varphi=\sin \varphi_{0} \cdot \cos v+\cos \varphi_{0} \cdot \sin v \cdot \cos \Delta \tag{52}
\end{equation*}
$$

and

$$
\begin{equation*}
\sin \Delta \lambda=\frac{\sin w \cdot \sin \Delta}{\cos \varphi} \tag{53}
\end{equation*}
$$

thus

$$
\begin{equation*}
\lambda=\lambda_{0}-l_{S}+\Delta \lambda . \tag{54}
\end{equation*}
$$

The angle of lowering the horizon will be (Fig. 6):


Fig. 5


Fig. 6

$$
\begin{equation*}
\cos x=\frac{R}{r} \tag{55}
\end{equation*}
$$

where:
$R$ - mean radius of the Earth,
$r$ - geocentric distance to the satellite according to (44).
Taking into account the radius of the Sun's disk we finally obtain:

$$
\begin{equation*}
h^{\circ}=-\left(x+0^{\circ} 50^{\prime} 4\right) \tag{56}
\end{equation*}
$$

The satisfaction of condition (51) is to be checked for a given flight, every station, and every position of the satellite at the points $Z$ and $M$ or $X$.

When performing computations on a digital computer the use of astronomical almanacs for determining the declination of the Sun and the equation of time would be rather troublesome. Because of that the following approximate formulae are here applied:

$$
\begin{align*}
& \delta^{\circ}=\varepsilon \cdot \sin \left[(T-82.3) \cdot \frac{2 \pi}{365.25}\right]  \tag{57}\\
&\left\{\begin{array}{l}
\eta_{1}
\end{array}=8^{m} \cdot \sin \left[(T-1.2) \cdot \frac{2 \pi}{365.25}\right]\right.  \tag{58}\\
& \eta_{2}=10^{m} \cdot \sin \left[(2 T+17.2) \cdot \frac{2 \pi}{365.25}\right] \\
& \eta=12^{n}-\left(\eta_{1}+\eta_{3}\right)
\end{align*}
$$

where $T$ - time in days, expressed as the succesive day of a year for which $\delta^{\circ}$ and $\eta$ are to be computed;
$\varepsilon$ - inclination of the ecliptic; we take $\varepsilon=23^{\circ} 26^{\prime} 5$.
The accuracy of the above formulae is quite satisfactory for the problem of computing the times of n. m. t. (n.e.t.) and sunrise (sunset).

## IV. Checking the computations

Let us assume that for a given tracking station we have the ephemevides (azimuths and altitudes) for two points, $Z$ and $X$ on the satellite's


Fig. 7 track, and the corresponding times $T_{Z}$ and $T_{X}$. The topocentric and geocentric distances to the points $Z$ and $X$ are also known, namely ez, $\varrho_{X}$, and $r_{Z}, r_{X}$, respectively. Fig. 7 illustrates the directions from the station to the satellite $I Z$ and $I X$ in the topocentric system of coordinates $N-E-Z$. For the purpose of checking the computations we shall try - using $T, h$ and $A$ - to determine the satellite's period; that period, compared with the given one is to verify the correctness of ephemerides for the points $Z$ and $X$. (Point $X$ is here chosen as an example. The method and the checking formulae are analogous for the set of points $Z$ and $M$ ). Let us introduce the following denotations:

$$
\alpha=\Varangle X I Z
$$

and $I X$, and $I Z$ - unit vectors of the directions $I X$ and $I Z$.
In the system of coordinates $\xi-\eta-\zeta$ that is turned with respect to the system $N-E-Z$ around the axis $Z$ so that the direction $I X$ lies in the plane $\xi \zeta$, the unit vectors $I X^{\prime}$ and $I Z^{\prime}$ have the following coordinates (Fig. 7):

$$
\begin{gathered}
\overline{I X^{\prime}}=\left[\cos h_{X}, 0, \sin h_{X}\right] \\
\overline{I Z^{\prime}}=\left[\cos h_{Z} \cdot \cos \left(A_{Z}-A_{X}\right), \cos h_{Z} \sin \left(A_{Z}-A_{X}\right), \sin h_{Z}\right]
\end{gathered}
$$

Hence the angle $a$ as an innerproduct of the unit vectors will be:

$$
\begin{equation*}
\cos a=\cos h_{X} \cos h_{Z} \cos \left(A_{Z}-A_{X}\right)+\sin h_{X} \sin h_{Z} \tag{59}
\end{equation*}
$$

Using the angle $\alpha$ and the distances $\varrho_{Z}$ and $\varrho_{X}$ we shall determine the lenght of the chord $X Z$ :

$$
\begin{equation*}
X Z^{2}=\varrho_{X}^{2}+\varrho_{Z}^{2}+2 \varrho_{X} \varrho_{Z} \cos \alpha \tag{60}
\end{equation*}
$$

which, in turn will serve for determination of the central angle $\beta$ between the geocentric radii of points $X$ and $Z$ :

$$
\begin{equation*}
\sin \frac{\beta}{2}=\frac{X Z}{2 r} \tag{61}
\end{equation*}
$$

where $2 r=r_{X}+r_{Z}$.
Hence the approximate period will be:

$$
\begin{equation*}
P^{\prime}=\frac{2 \pi}{\beta}\left|T_{X}-T_{Z}\right| \tag{62}
\end{equation*}
$$

This period should, with some tolerance, be coincident with the given period $P_{0}$ or, in case of low satellites, with the current period $P$ computed according to (12). The admissible difference between $P_{0}$ and $P^{\prime}$ depends upon the computational errors in $T, A$ and $h$ for the points $X$ and $Z$, and upon the eccentricity of the orbit. The greater the eccentricity the more non-uniform motion of the satellite (as it follows from the IInd law of Kepler); it moves most rapidly near the perigee and most slowly near the apogee.

The effect of computational errors in $T, A$ and $h$ can be disregarded since these quantities are computed on the URAL-2 computer with the accuracy of over 8 significant (decimal) digits. Assuming then that due to the accumulation of errors we obtain only 6 exact significant digits, the effected difference between the periods will be no more than of order of 0.1 min .

The permissible difference between the periods $P^{\prime}$ and $P$ will then depend first of all upon the quantity by which the greatest (or the least) velocity of the satellite will differ from the mean one; that difference depends upon the eccentricity of the orbit.

To estimate the quantity $\delta P_{\mathrm{adm}}=P_{\max }^{\prime}-P=P_{\min }^{\prime}-P$ the formula' for expansion of the true anomaly $v$ in series with respect to the mean anomaly $M$ is to be differentiated with respect to time. We shall consider
only two expressions of this expansion, it is to be pointed out, however, that further considerations are correct only in case of small eccentricities of the orbit (e.g. not exceeding 0.1)

$$
\begin{equation*}
v=M+2 e \sin M \tag{63}
\end{equation*}
$$

$$
\begin{equation*}
\frac{d v}{d t}=\frac{d M}{d t}+2 \cdot e \cdot \cos M \cdot \frac{d M}{d t} \tag{64}
\end{equation*}
$$

Formula (64) permits to define the current velocity of the satellite according to its position in the orbit that is determined by mean anomaly M. It is readily verified that $\frac{d v}{d t}$ reaches its exterme values if $M=0$ and $M=\pi$, i.e. at the perigee and at the apogee;

$$
\begin{equation*}
\left(\frac{d v}{d t}\right)_{\max }=\left(\frac{d v}{d t}\right)_{M=0}=\frac{d M}{d t}(1+2 e) \tag{65}
\end{equation*}
$$

and

$$
\begin{equation*}
\left(\frac{d v}{d t}\right)_{\min }=\left(\frac{d v}{d t}\right)_{M-a}=\frac{d M}{d t}(1-2 e) . \tag{66}
\end{equation*}
$$

Therefore, if the points $X$ and $Z$ will be e.g. near the apogee, then according to (62) period $P$ will be nearly

$$
P_{\max }^{\prime}=\frac{2 \pi}{\left(\frac{d v}{d t}\right)_{\min }}=\frac{2 \pi}{\frac{d M}{d t}(1-2 e)}
$$

However, $d M / d t$ is a so called mean daily motion, i. e.

$$
\frac{d M}{d t}=\frac{2 \pi}{P}
$$

We obtain then

$$
\begin{equation*}
P_{\max }^{\prime}=\frac{P}{1-2 e} \tag{67}
\end{equation*}
$$

Analogously we would obtain:

$$
\begin{equation*}
P_{\min }^{\prime}=\frac{P}{1+2 e} \tag{68}
\end{equation*}
$$

Hence, confining to the first powers of $e$ the admissible difference between the periods will be the following:

$$
\begin{equation*}
\delta P_{\mathrm{adm}}=\left|P^{\prime}-P\right|=2 P e \tag{69}
\end{equation*}
$$

In the case of satellite Alouette ( $e=0.003, P=105 \mathrm{~min}$.) we have

$$
\delta P_{\mathrm{adm}}=0.63 \mathrm{~min}
$$

## V. Floating chart

Logical design of the programme for computing the ephemerides for artificial earth satellites consists of two parts; their floating charts are given in Fig. 8 and 9.

Part I consists of the following steps:

1) Feeding in the input data, i. e. modified orbital elements, code numbers and coordinates of the tracking stations. This step includes also certain transformations of data, e.g. degree measures of angles into the are measures, etc. (boxes 1 and 2).
2) Computing the moments $T_{\text {n.m.t. }}^{\max }$ and $T_{\text {n.e.t. }}^{\min }$ for the region of stations with the use of the values $\lambda^{\text {min }}, \lambda^{\text {max }}$ and $\varphi_{\text {meen }}$ and from (9) and (10) (boxes 3 and 4).
3) Determining the coordinates of the reference point (boxes 5 to 12).
4) Computing the auxiliary quantities $l, \psi$ and $\Delta$, and moment $T_{50}$ (bozes 8 to 11 and 13 to 16 ). Substitution of zero on the auxiliary parametr $D^{\prime}$. Computing the moments $T_{\text {sunrise }}$ and $T_{\text {sunset }}$.
5) Substitution on the parametr $x$ unity if $\varphi_{0}>i$ and zero if $\varphi_{0}=i$ (boxes 18, 19 and 22). That value of $x$ indicates whether we have to deal with one or two branches of the orbit. In the latter case the current value of $x$ indicates whether the last branch considered was the ascending branch or the descending one.
6) Successive augmenting moments $T_{i 0}$ by quantity $D^{\prime}$ i. e. by the mean planar day of the orbit; by this means we obtain the moments $T_{i+1 ; 0}$. During the first course $D^{\prime}=0$ (boxes 20 and 23).
7) In the case $\varphi_{0}<i$ the individual flights determined by moments $T_{i 0}$ are to be considered in the following order:


Therefore, in this point of the programme the following values are to be substituted:

$$
\begin{aligned}
T_{i 0} & =T_{i 0}^{A} \\
\psi & =\psi^{4} \\
\Delta & =\Delta^{\Delta}
\end{aligned}
$$

(box 21)


Fig. 8. Floating cha rt - part I


Fig. 9. Floating chart - part II
8) Since the ephemerides are to be computed from a given initial epoch $T_{\text {begln }}$ then it is necessary to investigate whether $T_{i 0}$ is greater then $T_{\text {begin }}$ (box 24).
9) Box 25 illustrates the investigations necessary to see whether the moment $T_{i 0}$ under consideration is not greater than the final epoch $T_{\text {end }}$. In the case $T_{i 0} \leqslant T_{\text {end }}$ the computer goes on to further computa-
tions; if, however, $T_{i 0}>T_{\text {ond }}$ the signal for ending the computations occurs and the machine stops.
10) According to (7) we sball now consider the moments $T_{i j}$. First, the parameter $y$ determining the actual value of index $j$ should be set to zero. If

$$
\begin{array}{lll}
y=0 & \text { then } & j=1 \\
y=1 & & j=2 \\
y=2 & j=2
\end{array}
$$

(boxes 26 and 27).
11) Further computations are performed only for those moments $T_{1 j}$ those satisfy conditions (8a) or (8b). In this case the computer goes on to perform the computations in the II-nd part of the floating chart (boxes 28 to 31 ).
12) If, however, the given moment $T_{i j}$ is rejected as non-satisfying the conditions (8) or after having performed the computations in the II-nd part, the parameter $y$ is to be augmented by 1 (box 32) what is followed by the investigation whether the new value of $y$ exceeds 2 (box 33).
13) The negative result of this investigation proves that not all moments $T_{i j}(i=$ const) corresponding to three successive flights of the satellite near the reference point have been used. Therefore, the last moment considered is to be augmented by the period $P$ (box 34) followed by checking for satisfaction of conditions (8) by this new moment.
14) In case of exhausting all moments $T_{i j}(i=$ const) 1 is to be subtracted from the value of parameter $x$ (box 35 ).
15) The new value of $x$ equals zero if the last branch considered is the ascending one. If, however, the branch is the descending one and if $\varphi_{0}=i$, the new value of $x$ equals - 1 (box 36).
16) If the result of investigation $x=0$ in box 36 is positive, the following substitutions should be made:

$$
\begin{aligned}
T_{i 0} & =\pi_{i 0}^{D} \\
\psi & =\psi^{D} \\
\Delta & =\Delta^{D}
\end{aligned}
$$

We return then to box 25 , i. e. to investigate whether $T_{i 0} \leqslant T_{\text {end }}$ (box 37). If, however, $x \neq 0$ then for $D^{\prime}$ we substitute $24^{h}+D$ (mean planar day) and return to box 18 where, according to the result of investigation $\varphi_{0}=i$ we substitute $x=1$ or $x=0$ (boxes 19 and 22); the new value of $T_{i 0}$ is then obtained (boxes 20 and 23).

The operation of the I-st part of the programme consists then in generating the successive moments $T_{i j}$ and eliminating the ones not satisfying the conditions (8).

The moments $T_{i j}$ that satisfy these conditions make the basis for computing the ephemerides for specified stations at given flight of the satellite. This problem is included in part II which is also to be divided into separate steps:
17) Computing the moment $T_{p}$ in which the satellite is at the perigee for the last time before given $T_{i j}$, the moment at which the satellite crosses the reference latitude ( $T_{s}$ ), and the longitude of the crossing point ( $l_{S}$ ) (boxes 1 to 6 ).
18) Further computations are to be performed with respect to each station, the coordinates of which are included in the input data. For this purpose a number of repetitions of the same fragment of the programme should be declared according a number of stations - each repetition for data of each station (box 7).
19) Computing the difference betwoen the longitude of the station and the longitude of the point at which the satellite crosses the reference latitude, i. e. between longitudes of point $I$ and $S$ (box 8).
20) Solving the spherical triangle $S B I$, i. e. computing the angles $a$ and $\beta$, and the side $m$ (box 9). Solving the rectangular spherical triangle $S I Z$, i. e. computing $d_{Z}$ and $w_{Z}$ (box 10).
21) Computing the horizontal altitude $h_{Z}$ and distances: geocentric $r_{z}$ and topocentric $\varrho_{z}$ of point $Z$ (box 11).
22) Investigating whether $h_{Z} \geqslant 22^{\circ}$ (box 12). If the result of this investigation is negative then the computer goes on to investigate whether all the stations are used during a given flight (box 39). Accordingly, the computer either starts the computations for the next station (return to box 8) or, in the case when all the stations are exhausted, it comes back to part I where a new flight is to be generated.
23) If $h_{Z} \geqslant 22^{\circ}$ further computations for a given observatory are to be performed, namely computing the azimuth of point $Z\left(A_{Z}\right.$, box 13), and the moment at which the satellite crosses point $Z\left(T_{Z}\right.$, box 14$)$.
24) Boxes 15 and 16 illustrate the investigation whether at moment $T_{Z}$ a given station is at a night time. In the analogous way the investigations are to be made for illumination of the satellite by the Sun (boxes 17 and 18), i. e. to see whether condition (50) is satisfied. Similar investigations in part I are to be regarded as the approximate ones since they refer not to a definite station and moment $T_{z}$ but to the geographical region including all stations, and to time interval ( $T_{i j}-\frac{1}{2} P_{0} ; T_{i j}+\frac{1}{2} P_{0}$ ). The purpose of those investigations was to eliminate the flights during which the satellite is undoubtedly not visible from the region of stations.

The non-eliminated flights include then the flights during which the satellite may not be seen from one or more stations.
25) In the case when the results of above investigations indicate that the configuration satellite-station-Sun at moment $T_{z}$ makes the optical observations not possible, the computer (like in item 22) either continues to perform the computations for the next station or, in the case all stations are exhausted, it returns to part I of the programme.
26) If, however, a position of the satellite at moment $T_{Z}$ is convenient for optical observations from a given station, then the number of this station and the values $T_{z}, A_{z}$ and $h_{z}$ are to be derived (boxes 19 and 29); the machine goes then on to perform analogous computations for point $M$ (boxes 21 and 22).
27) If $h_{M} \geqslant 15^{\circ}$ (box 23) the moment of crossing the meridian ( $T_{M}$, box 24) is to be computed, then follows the investigation to see whether the satellite is still out of the Earth's shadow (boxes 25 and 26), and time interval between crossings the points $Z$ and $M$ is more than 2 min . (box 27 ).
28) In the case when positive results of the above investigations are obtained the ephemeris for point $M$ is to be derived (box 28), checking computations are then to follow (box 38).
29) The negative result of any of these investigations proves that the satellite cannot be seen at the point $M$ and therefore the ephemeris for point $X$ should be computed. After having computed $h_{X}, A_{X}$ and $T_{X}$, with the use of appropriate auxiliary quantities (boxes 29 to 33 ) the machine goes on to investigate illumination of the satellite by the Sun (boxes 34 and 35).
30) If these investigations indicate that the satellite is out of the Earth's shadow the ephemeris for point $X$ is to be derived (box 36); checking computations are then to follow. If, however, the satellite is not illuminated by the Sun, a new value is to be substituted for $w_{X}$ (box 37), and the computations are to be repeated starting with box 30.
31) After having computed the ephemerides for the satellite (for given station and flight) at two points on its track, the next step is to check the computations. The result of checking is then derived, and the machine goes on to perform computations for the next station or, in the case all stations are exhausted, to generate a new flight in part I (box 39).

## VI. Data and results

## 1. Preparing the input data

All the input data of this programme are to be fed into the computer from the teleprinter tapes through the high-speed readers. Preparation of the tapes consists in punchnig separate decimal numbers in a certain
order which is determined by the appropriate distribution of the numbers on the teleprinter sheet.

In what follows we give an example of printing the input data on the teleprinter sheet:
a) modified orbital elements:

62491
$\begin{array}{llllllllll}16 & 6 & 7 & 25.76 & 80.47 & 104.99 & -7.84 & 37.85 & -0.187 \\ 105.412 & 0 & 0.00284 & 4580.1 & 0 & 23 & 6 & 31 & 6 & \end{array}$
The number 62491 is here a code number of the satellite (the last two digits of a year in which the satellite was placed in the orbit, the successive number of the satellite in this year, and the denotation of the object, e. g. 1 - satellite, 2 - the last stage of the rocket, etc.). Further numbers are given in succession and units indicated in section II, item 1, e. g.:

$$
\begin{aligned}
& T_{0}=16 \text { June } 7^{\mathrm{h}} 25^{m} \cdot 76 \quad(D T) \\
& i=80^{\circ} .47 \text { etc. }
\end{aligned}
$$

b) data on the stations:

5
$\begin{array}{lll}1151 & 1.3639 & 53.7536\end{array}$
$1154 \quad 1.1253 \quad 52.3973$
$1155 \quad 1.4117 \quad 52.2183$
$1160 \quad 1.4024 \quad 52.0984$
$1162 \quad 1.3306 \quad 50.0644$
The number 5 denotes a number of stations in a given group and the separate columns include successively:

- code numbers of stations,
- longitudes in hours,
- latitudes in degrees.


## 2. Deriving the result on the lineoprinter

The programme makes possible to derive the ephemerides either on the line-printer or on the paper tape punch.

In what follows we give an example of the results derived on the line-printer.
$+11510000+04$
$+60000000+01$
$+28000000+02$
$+22000000+02$
$+37182397+02$$\quad$ - station code number

| $+28738867+03$ | - azimuth $A_{z}$ in degrees |
| :--- | :--- |
| $+70065434+02$ | - horizontal altitude $h_{Z}$ in degrees |
| $+60000000+01$ |  |
| $+28000000+02$ | moment $T_{M}\left\{\begin{array}{l}\text { month } \\ \text { day } \\ \text { hours } \\ \text { minutes }\end{array}\right.$ |
| $+22000000+02$ | - azimuth $A_{M}$ |
| $+00000000+00$ | - hor. altitude $h_{M}$ |
| $+37298236+02$ | - result of checking $\left(P-P^{\prime}\right)$ in minutes |
| $-34707914+00$ | - checking symbol |

(Decimal number derived on the line-printer consists of the decimal fraction and exponent parts. They are to be read in consideration of the following rule:
decimal number $=$ decimal fraction part $\cdot 10^{\text {exponent part }}$,
i. e. for example $+28738867+03 \equiv+287.38867$ ).

## 3. Deriving the reaults on the paper tape punch

In this case when running the obtained panched paper tape through the teleprinter we get ready telegrammes about the satellite's flight for each station, that are coded in the international code. The telegrammes are of the following form (for example):

SATAT 1151X 62491 87X26 1948X 29837 1950X 33430 千
The successive 5 -symbol groups have the following meaning:

- code word indicating that the telegramme includes the ephemeris;
- code number of the station for which the ephemeris is given in the telegramme; (letter $X$ is here of no significance);
- code number of the satellite:
- first two digits make the sum of all the remaining digits in the telegramme (for checking purposes); last two digits mean the day of a month for which the ephemeris is given;
- $T_{Z}$ in hours ( 2 digits) and minutes ( 2 digits) of the universal time;
- $\boldsymbol{A}_{\boldsymbol{Z}}$ in degrees ( 3 d degits ) and $h_{Z}$ in degrees ( 2 digits);
- $T_{M}$ or $T_{X}$ in hours and minutes of $U T$;
$-A_{M}$ or $A_{X}$ and $h_{M}$ or $h_{X}$ in degfrees.
The sign + at the end of a telegramme means that the quantity $\delta P$ as a result of checking computations is within the admissible limits.


## VII. Final remarks

1) The programme for computing the ephemerides for artificial earth satellite is prepared in the autocode KLIPA (Kod Liczbowej Interpretacji Parametrów Adresowych - Code of Numerical Interpretation of Addressed Parameters) elaborated at the Computation Centre of Polish Academy of Sciences. The programme include about 1500 instructions, pseudo-instructions, and decimal parameters.
2) Time for computing the ephemerides for a period of $m$ weeks and for $n$ tracking stations is:

$$
T \cong(10 \mathrm{sec}+n \cdot 20 \mathrm{sec}) \cdot m
$$

This quantity does not include time required for feeding in the programme and the input data (ca 1 min. ), it includes, however, the time needed for deriving the results on the paper tape punch.
3) As previously mentioned, the programme is applicable to positive orbit inclinations and Northern latitudes of stations.
4) The observations of satellite Alouette made chiefly at the station of the Warsaw University confirm that the discussed method for compating the ephemerides is correct.

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