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Mathematical foundations of information storage and retrieval

Part 3

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Abstract • Содержание • Streszczenie

We show how to incorporate within our formalism hierarchical aspects of information retrieval.

Математическое описание процесса поиска
и хранения информации. Третья часть

В работе показываем способ, как можно представить иерархические аспекты поиска и переработки информации.

Matematyczne podstawy wyszukiwania i gromadzenia
informacji. Część 3

Pokazujemy, w jaki sposób w naszym formalizmie można ująć hierarchiczne aspekty wyszukiwania i przechowywania informacji.

§ 1. Hierarchization

Definition 1.1. Let R, S be equivalences on X. We say that $R < S$ iff $R \subseteq S$ i.e.

$$(\forall x)(\forall y)(xRy \longrightarrow xSy)$$

It is clear that $<$ is partial ordering

Definition 1.2. Let R be an equivalence on A and S an equivalence on A/R. We define a relation $S * R$ on A as follows:

$$xS * Ry \longleftrightarrow x/R \; S \; y/R$$

Lemma 1.1. $R < S * R$

Proof: Assume xRy. Then $x/R = y/R$ and since S is reflexive we get $x/R \; S \; y/R$ i.e. $xS * Ry$.

Lemma 1.2. Assume $S*(T*R)$ is defined. Then $(S*T)*R$ is defined and $S*(T*R) = (S*T)*R$.

Proof: Assume T is defined on A/R and S defined on A/R/T. Then $S*T$ is defined on A/R and so $(S*T)*R$ is defined.

Let $xS*(T*R)y$. Then $x/T*R \; S \; y/T*R$. Having in mind that $x/T*R$ consists of all those y/R which are (with x/R) in relation T we find that

$$x/R \;\; S*T \; y/R$$

which is desired result.

Lemma 1.3. If $S < R$ then there is T such that

$$R = T * S$$

Proof: We define $x/S \; T \; y/S$ iff xRy.
It is enough to prove that T is equivalence. Clearly T is reflexive and symmetric. If $x/S \; T \; y/S \; T \; z/S$ then xRy and yRz and so xRz i.e. $x/S \; T \; z/S$.
As we remember $A = \bigcup_{i=I} A_i$ was a decomposition. Since decomposition is nothing else but family of equivalence classes of some equivalence, we have relation R_I which determines the decomposition $\{A_i\}_{i \in I}$. We may identify I with A/R_I.

Definition 1.3. Let $S < R_I$, $\mathfrak{S}$ be i.s.r. system, $\mathfrak{S} = \langle X, A, I, U \rangle$
We define $\mathfrak{S}/S$ as follows:

$$\mathfrak{S}/S = \langle X, A/S, I, U/S \rangle$$

where $U/S(a/S) = \bigcup \{U(b) : bSa\}$
Clearly $\mathfrak{S}/S$ determines language $\mathcal{L}(A/S)$. The unique relation T such that $R_I = T * S$ comes into the axioms of corresponding theory as follows:

$$\sum_{b/S \; T \; a/S} c_{b/S} = V$$

Let us now enrich the language $\mathcal{L}(A)$ by constants $c_{a/S}$.
Our theory is also enriched as follows

$$(*) \qquad c_{a/S} = \sum_{bSa} c_b$$

In this way we obtain hierarchical i.s.r. system of rank 2 generated by S as follows: $\mathfrak{S}_S = \langle X, A \cup A/S, I, U \cup U/S \rangle$

(Notice that this is nothing else but $\mathfrak{S} \oplus \mathfrak{S}/S$! ! !)

Lemma 1.4. $\mathfrak{S}_2$ satisfies (*)

Full power of the operation $\oplus$ is seen when we have a sequence of relations:

$$S_1 < S_2, \ldots, < S_n < R_I$$

Definition 1.4. Under above assumptions we define

$$\mathfrak{S}_{S_1, \ldots, S_n} = \mathfrak{S} \oplus \left(\bigoplus_{i=1}^{n} \mathfrak{S}/S_i \right)$$

Let $T_1, \ldots, T_{n-1}$ be relations such that $S_{i+1} = T_i * S_i$

Lemma 1.5. $\mathfrak{S}_{S_1, \ldots, S_n}$ satisfies the following formula

$$c_{a/S_{i+1}} = \sum_{b/S_i \; T_i \; a/S_i} c_{b/S_i}$$

Proof: It is clear that it is enough to give the proof for the case $S_1 < S_2 < R_I$, $S_2 = T * S_1$.

Indeed, for $a \in A$ $c_{a/S_2} = \sum_{b \; S_2 \; a} c_b$ and $c_{a/S_1} = \sum_{b \; S_1 \; a} c_b$

Since however $S_1 < S_2$ we have, for $a, b \in A$.

$$c_{a/S_1} \leqslant c_{a/S_2}$$

Using idempotence law we get

$$c_{a/S_2} = \sum_{b \; S_2 \; a} c_{b/S_1}$$

But this exactly reduces to the desired equation.

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