Modelling Spot Prices on the Polish Power Exchange

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Abstract. In this paper a model for the dynamics of the Polish Power Exchange electricity spot prices is proposed. The model describes most important quantitative features of evolution of the price index which are characteristic to the Polish market.
The dynamics of the electricity spot prices is governed by a mean-reverting jump-diffusion stochastic process with mixed-exponentially distributed jumps. Estimation of the model's parameters is based on historical data. The model may be precisely calibrated to quoted forward contracts, making use of the analytical formula for a forward price.
In the paper valuation of plain vanilla options on the electricity spot price via the Monte Carlo method is also presented.

1 Introduction

The establishment of the Polish Power Exchange (POLPX) was a result of an implementation of the new law in Poland in April 1997, whose one of the most important assumptions was to liberalize the Polish energy market. At that moment the process of restructuring the energy sector was conducted in the majority of European countries. The main purposes behind the reforms were to disassociate electricity, as a tradable commodity, from its transmission services and to establish a market for electricity generators, energy suppliers, companies involved in energy trading and industry clients.

The POLPX began to operate in December 1999. Within six months the electricty spot market started to run. In this way, bilateral contracts gained a benchmark pricing index. The 2000s decade is a period of a very fast development of the POLPX: a property rights market, a register of certificates of origin for the electrical power generated from renewable sources and produced in cogeneration, a spot market for CO_2 emission certificates and finally an electrical power derivatives market were launched.

The trading volume on all electricity markets on the POLPX in 2014 was equal to 186.8 TWh which is 119.4% of the energy generation and 117.7% of energy consumption in Poland in this year. Currently, there are 66 members of the electricity spot market.

Econometrical patterns, which are typical for most of electricity spot prices time series, are present also while analysing data coming from the POLPX's index. These include: daily, weekly, yearly seasonality, abrupt, usually unexpected jumps and mean-reversion - when the prices are in a spike regime and the extraordinary market situation (caused e.g. by a failure of a transmission network, outage of power plants, a sudden decrease or increase in temperature, low levels of water or droughts, changing possibilities of exploitation of renewable energy sources) finishes, the prices immediately recur to the former, normal level.

In the paper the dynamics of the spot electrical energy prices is modelled by a continuous-time stochastic process which takes into consideration the aforementioned features of prices, i.e. by a mean-reverting jump-diffusion with mixed-exponential jump size distribution. Our model belongs to the class of one-factor models which are characterized by their good matching to data, existence, in many cases, of analytical solutions to various considered problems (e.g. formulas for forward prices) and existence of numerous approximation methods (e.g. for pricing options, etc.). The basic, initial model was introduced by [1] in which the authors decomposed the signal to a seasonality and a mean-reverting to zero diffusion process. However, the authors did not take the possibility of jumps in prices into account.

In [2] a mean-reverting jump-diffusion model with normally distributed jumps was introduced. There is a possibility of deriving an analytical formula for a forward price. Unfortunately, the distribution of jumps on the POLPX's time series of spot electricity prices is not unimodal. What is more, the authors did not explain how to assure that after filtering the jumps from the series, the returns have normal distribution. Another reason for which the model cannot be applied to the Polish market is that the subject of calibration of the model to quoted on the market forward contracts, using the analytical formulas for the forward price, was not raised. Other authors in [3] also noticed that the usage of the Normal distribution of spikes has an effect of overestimation of skewness and kurtosis.

A very interesting approach was presented in a threshold model [4] where the mean-reverting diffusion was combined with a time-inhomogeneous Poisson process of a truncated exponential distribution of jumps. Moreover, to allow for downward, reverting to the mean jumps, the authors introduced a characteristic function indicating the sign of a jump which depends on a current value of a price. The proposed switching threshold is a constant positive spread over a seasonality.

Notwithstanding, the model has some drawbacks as well. There is no possibility to obtain the analytical formula for a forward price. Additionally, [3] critisizes the choice of the truncated exponential distrbution due to the fact that it disallows for big jumps exceeding the fixed threshold. There is also noted that two consecutive jumps of the same sign are impossible to occur and after estimating the model's parameters, the mean-reversion's parameter turned out to be higher than expected for a base signal and smaller than required to dampen a spike.

Another interesting subclass of one-factor models are regime-switching models. Markov models are very popular nowadays and also in the field of electrical energy prices modelling they are widely applicable. The reason is that one can define separate forms of dynamics for all substantially different ranges of prices values, usually there are three of them: two spike regimes when the prices achieve anomalous values after the upward and downward jumps, and a normal, base regime. There is also a transition matrix which links the regimes by indicating how much the transition from one state to another is probable. For details, see [5, **?**,**?**].

An alternative to all above-mentioned approaches may be a model in which a diffusion generated by a Wiener process is superseded by very frequent and small jumps (representing typical, daily movements of prices) generated by a Levy process of infinite activity. The Levy process is also responsible for big jumps in prices (substitution for the Poisson process). The model was described in [8].

In Section 2 the dynamics of our custom-made model for the Polish market is enunciated. The rest of the paper is organised as follows. Section 3 familiarizes the reader with historical data chosen for analysis. In Section 4 the method of adjusting seasonality to the historical time series is written up in details and also one becomes acquainted with the algorithm of detection of spikes in prices. The course of a process of the parameters estimation is comprised in Sections 5 and 6. In Section 7 the discretization of the continuous-time dynamics, as well as the comparison of the simulated this way trajectories with the historical series (tests for a goodness of fit) are performed. Section 8 demonstrates the form of the analytical forward price and introduces the notions of the market prices of risks. The next paragraph includes the results of option pricing. The last section concludes.

2 The model of the Polish Power Exchange spot prices

Let us now describe the model of the spot prices which reflects the distinctive features of the Polish energy market. The dynamics is governed by a mean-reverting jump diffusion process with the jump size distribution, the idea of which is borrowed from [9]. In that paper asset (not commodity) prices were considered, but inasmuch as the distribution of returns has fatter tails than the normal distribution, the authors decided to add a compound Poisson process component, jumps of which are sampled from the mixed-exponential distribution. This distribution can approximate any distribution with respect to weak convergence as closely as possible and this fact was an inspiration to use this jump distribution in our model. Existence of jumps in case of electricity spot prices trajectories is definitely more pronounced than in case of any other market's prices. Therefore, flexibility in fitting a theoretically described distribution to a dataset of jumps is an added value.

A relative simplicity of the model's formulation results in capability of deriving the closed-form forward price. This in turn enables to calibrate the model to the quoted forward contracts in a very precise way.

We start with the decomposition of the spot price process S_t:

$$S_t = \exp(g(t) + X_t), \tag{1}$$

$$dX_t = -\alpha X_t dt + \sigma dW_t + dJ_t, \tag{2}$$

where α and σ are constants, $(W_t)_{t\in\mathcal{T}}$ is a Wiener process, $(J_t)_{t\in\mathcal{T}}$ is a compound Poisson process of the form

$$J_t = \sum_{i=1}^{N_t} Z_i, \;\; t \in \mathcal{T},$$

with constant intensity λ, Z_i are i.i.d. jump magnitudes of translated mixed-exponential distribution, i.e. with density

$$f(z) = q_d \sum_{i=1}^{m} q_i \xi_i e^{\xi_i (z-m_d)} \mathbb{1}_{\{z<m_d\}} + p_u \sum_{j=1}^{n} p_j \eta_j e^{-\eta_j (z-m_u)} \mathbb{1}_{\{z>m_u\}}, \tag{3}$$

where

$$q_d, p_u \geq 0, \;\; q_d + p_u = 1, \;\; q_i, p_j \in (-\infty, \infty), \;\; \sum_{i=1}^{m} q_i = \sum_{j=1}^{n} p_j = 1, \;\; \xi_i > 0, \eta_j > 1.$$

q_d and p_u are the probabilities of negative and positive jumps, respectively. $m_d < 0$ is a minimal (with respect to the absolute value) value of negative jumps, $m_u > 0$ is a minimal value of positive jumps. A necessary condition for $f(z)$ to be a density function is

$$q_1, p_1 > 0, \;\; \sum_{i=1}^{m} q_i \xi_i \geq 0, \;\; \sum_{j=1}^{n} p_j \eta_j \geq 0.$$

One of possible sufficient conditions is

$$\sum_{i=1}^{k} q_i \xi_i \geq 0, \;\; \sum_{j=1}^{l} p_j \eta_j \geq 0$$

for all $k \in \{1, \ldots, m\}$, $l \in \{1, \ldots, n\}$. A special case of the mixed-exponential distribution is a hyperexponential distribution, when all parameters q_i and p_j are nonnegative.

The separation from zero of the support of the density function is caused by the fact that either positive or negative jumps are extreme events, therefore highly greater than zero with respect to absolute value.

Using the Ito lemma, one obtains that S_t follows the stochastic differential equation

$$dS_t = \alpha(\rho(t) - \ln S_t)S_t dt + \sigma S_t dW_t + S_t(e^Z - 1)dN_t, \tag{4}$$

where

$$\rho(t) = \frac{1}{\alpha}\left(\frac{dg(t)}{dt} + \frac{1}{2}\sigma^2\right) + g(t).$$

3 Historical data

Data chosen for estimation of the model's parameters comes from the POLPX's IRDN spot index and covers the period of October 2011 – September 2015 (1443 historical prices). It is important to note here that by a spot price we mean a

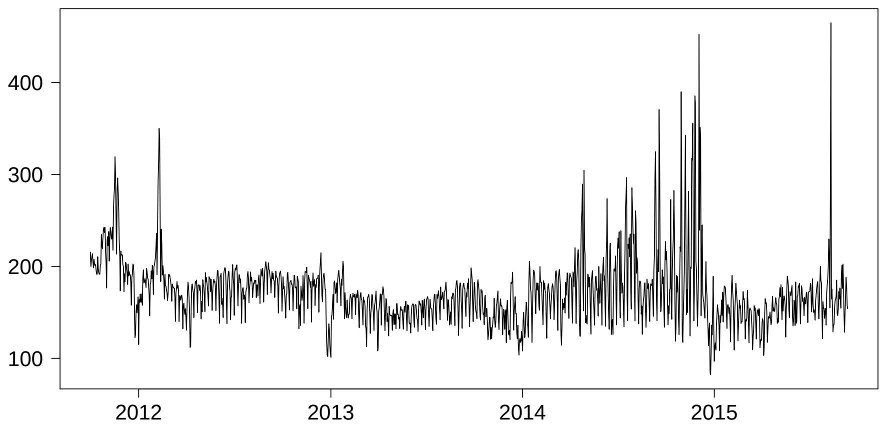

Fig. 1. Spot prices in PLN/MWh

weighted (by volume) average price of daily transactions – a standard day-ahead reference index for contracts with delivery of energy during the whole upcoming day.

At first sight one can state that prices undergo some yearly and weekly (prices on Sundays are decidedly smaller than on other days) seasonal movements and that from time to time a spike occurs.

4 Seasonality matching and filtering of spikes

In this paper an original, robust method of deseasonalisation is proposed. Before all, a very important aspect to consider is that after the logarithm transform and deseasonalisation of prices (see eq. (1)) combined with the filtrating of jumps, the remaining residue is modelled by the zero-mean-reverting diffusion process which has normally distributed increases. It means that the way of seasonality matching and filtering of spikes must not on any account be arbitrary. Our idea is to perform spikes filtering so as to maximize a p-value of the appropriate statistical test for normality of the aforementioned increases.

The deseasonalisation itself is divided into several stages.

4.1 Downward spikes appearing on holidays

The Polish market has an attribute that if a day is a national holiday, then a negative spike takes place. These spikes are removed from the series as a first part of deseasonalisation. Every spike's value is replaced with a mean of 5 preceding and 5 following prices. There are 12 such deterministic downward spikes each year.

4.2 Matching of weekly and yearly oscillations

The weekly seasonality is computed as means of logarithms of prices of all days within a week. Afterwards, these values are subtracted from the log-index, but days of the holidays are excluded from this procedure. The yearly fluctuations are found by adjusting, by a nonlinear least-squares method, a one-year periodic, sinusoidal function of the form

$$a + bt + \sum_{k=1}^{3} c_k \sin\left(\frac{2k\pi t}{365}\right) + d_k \cos\left(\frac{2k\pi t}{365}\right).$$

The fitted this way function is shown in Figure 2.

4.3 Spikes filtering

Filtering of spikes is performed by an iterative procedure: in the first step all jumps which absolute value exceeds some predefined threshold, for instance three times the standard deviation of the deseasonalised log-returns, are removed from the series. In the next step the same action is made, but this time the standard deviation is calculated basing on the thinned series of returns. New jumps are filtered and deleted and the process continues until in some iteration no jumps are found.

The most important aspect of this method is to fix the threshold so as to maximize the p-value of the Anderson-Darling normality test for the deseasonalised, and with deleted jumps, log-returns – the assumptions of the model must be fulfilled. For our data the threshold turned out to be $2.45s$, where s is the standard deviation of the series obtained in each step of the described procedure. The maximized p-value is equal to 0.113. There is no evidence to reject the null hypothesis of the log-returns normality at the 10% significance level. A similar idea, referring to the shape of a seasonality function, was applied in [10].

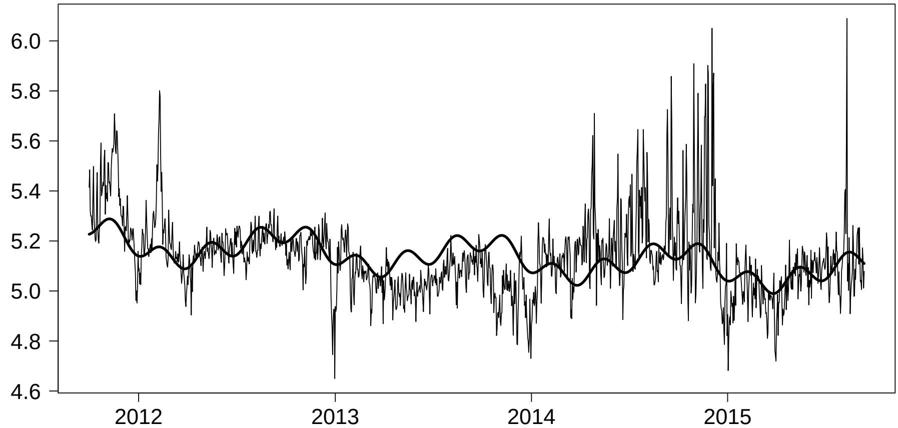

Fig. 2. Annual sinusoidal function fitted to the partially deseasonalised log-price series

The choice of the Anderson-Darling normality test is dictated by its one of the best capabilities of detecting most departures from normality, cf. [11].

After filtering of spikes, their values in the series are replaced by a mean of 5 preceding and 5 following prices.

4.4 Making seasonality independent from the spikes occurrences

If the seasonality was matched basing upon the raw historical data, then this estimation would be biased by the presence of jumps. To counteract this problem, we propose the following procedure:

1. logarithmize the input series of prices and remove holidays downward spikes (cf. Subsection 4.1),
2. eliminate the rest part of seasonality, i.e. weekly and yearly oscillations (cf. Subsection 4.2),
3. filter out and remove spikes (cf. Subsection 4.3),
4. add the seasonality fitted in point 2. to the deseasonalised and bereft of spikes series and then once again perform the (this time robust to spikes) deseasonalisation described in point 2.

Obtained this way seasonality is not influenced by the magnitudes of jumps in prices and thus should be used as a part of the formula (1) to achieve the historical realization of the process (2). A similar technique was adapted in [12].

It is worth to see this aggregate form of the seasonality applied for the upcoming years, see Figure 3. The seasonality in some magnification, around Christmas, is shown in Figure 4.

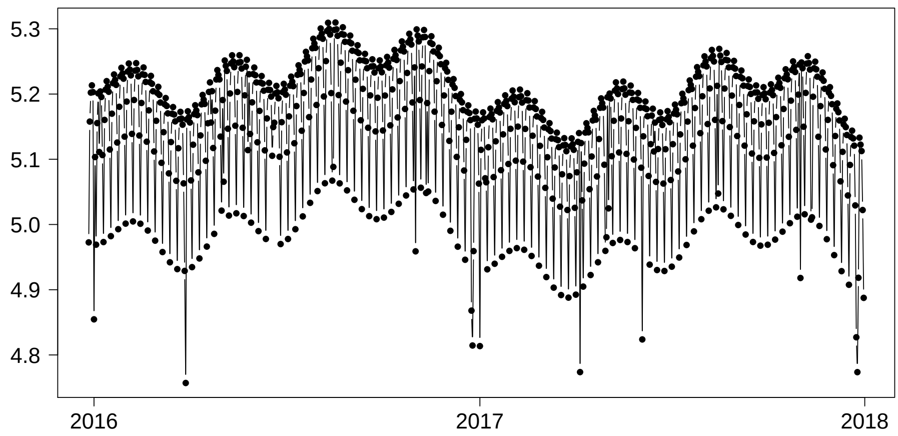

Fig. 3. The overall seasonality function in logarithmic scale in PLN/MWh

5 Estimation of the driving process's parameters

After the deseasonalisation and removal of spikes from the log-series, one may proceed to estimation of the jump-diffusion's parameters. The volatility σ from the equation (2) is estimated as a mean of the rolling standard deviation of the time-scaled increments $\frac{P_i - P_{i-1}}{\sqrt{t_i - t_{i-1}}}$ (see [13], formula 3.10):

$$\sigma(t_k) = \sqrt{\frac{1}{m-1} \sum_{i=k-m+1}^{k} \left(\frac{P_{i+1} - P_i}{\sqrt{t_{i+1} - t_i}} - \frac{1}{m} \sum_{j=k-m+1}^{k} \frac{P_{j+1} - P_j}{\sqrt{t_{j+1} - t_j}} \right)^2},$$

where P is the deseasonalised and devoid of spikes log-price index, $m = 30$, $M = 1305$ (after removing of jumps there are 1305 log-returns), $k \in \{m, \ldots, M\}$. For all $i \in \{1, \ldots, 1306\}$ $t_{i+1} - t_i = \frac{1}{365}$. The estimated value $\sigma = 1.14$.

Determination of the mean-reversion's velocity α is based on the deseasonalised log-prices, but in the presence of spikes. One has to regress the deseasonalised log-prices series bereft of its first element versus the deseasonalised log-prices series without its last element, which is a direct cause of the discretized form (see details in Subsection 7.1) of the equation (2):

$$X_{t_k} = e^{-\alpha \Delta t} X_{t_{k-1}} + \rho_{t_k},$$

where ρ_{t_k} is the sum of integrals of the Wiener process and the compound Poisson process between times t_{k-1} and t_k. The value of the regression coefficient $e^{-\alpha \Delta t}$ is significantly different from zero – the speed of mean-reversion achieved this way equals $\alpha = 0.3$.

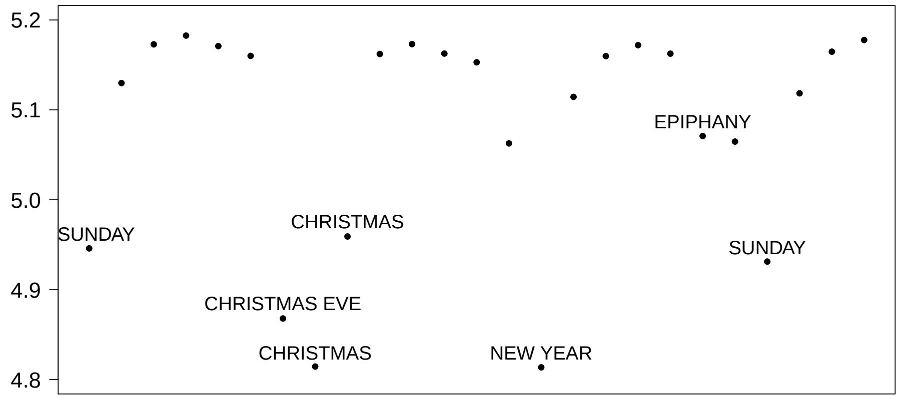

Fig. 4. Seasonality function around Christmas 2016

The results of the augmented Dickey-Fuller test applied for the deseasonalised log-prices indicate that there is no unit root in our time series data – the mean-reversion is indeed present.

6 Evaluation of the jump-size distribution's parameters

To filter jumps for the purpose of the jump-size distribution's parameters estimation, we use the algorithm described in Subsection 4.3, but a salient modification is necessary – some of the filtered jumps are mean-reversions of the process and thus have to be excluded from the analysis. Accordingly, if there are two or three consecutive jumps and the last one is of opposite sign, it is regarded as a mean-reversion. 137 returns are classified as jumps by the filtering algorithm and out of them 43 are assessed as mean-reversions, yielding the yearly frequency of the Poisson process $\lambda = (137 - 43)/1442 \cdot 365 = 23.8$. Counting downward and upward jumps brings $q_d = 0.32, p_u = 0.68$. The minimal sizes of negative and positive jumps are equal to $m_d = -0.152, m_u = 0.148$, respectively.

The remaining parameters are estimated by the maximum likelihood method – see Table 1 (in the density function specification (3) we take $m = n = 2$, which is a compromise between the accuracy and the number of parameters to be evaluated).

The parameters q_1, q_2, p_1, p_2 are all positive, so that the jump-size distribution turns out to be hyperexponential, a special case of the mixed-exponential distribution. Figure 5 illustrates the adjustment of the density to the empirical distribution of filtered jumps.

q_1	q_2	ξ_1	ξ_2	p_1	p_2	η_1	η_2
0.06	0.94	1.78	21.78	0.64	0.36	5.79	40.66

Table 1. Estimated parameters of the mixed-exponential jump size distribution

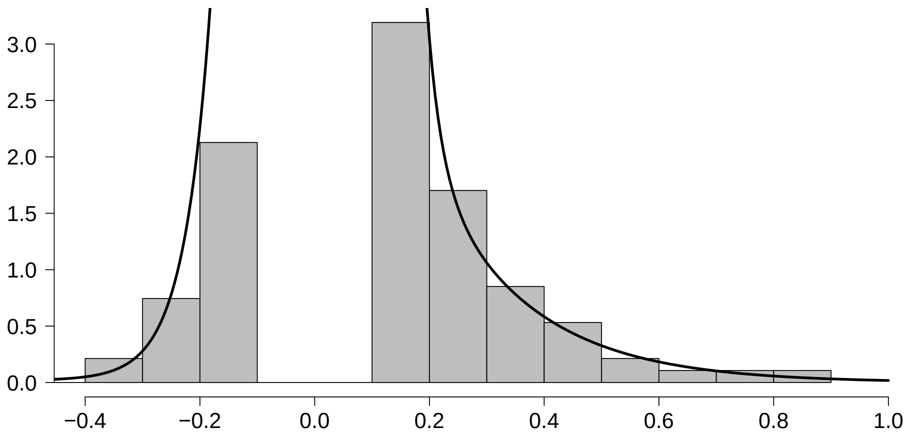

Fig. 5. Mixed-exponential distribution fitted to the empirical distribution of jumps

7 Simulation of the spot prices and tests for the trajectories

7.1 Discretization of the process

Lemma 1. *Let X_t follow the equation (2) and may $0 \leq s \leq t$, $t \in \mathcal{T}$. Then*

$$X_t = e^{-\alpha(t-s)} X_s + \int_s^t \sigma e^{-\alpha(t-u)} dW_u + \sum_{s<u\leq t,\ \Delta N_u \neq 0} e^{-\alpha(t-u)} \Delta J_u. \quad (5)$$

Moreover,

$$\int_s^t \sigma e^{-\alpha(t-u)} dW_u \sim N\left(0, \sigma \sqrt{\frac{1-e^{-2\alpha(t-s)}}{2\alpha}}\right). \quad (6)$$

Proof. We refer the reader to [14].

Hence, the discretized dependency between the consecutive "daily" values of the process X_t is of the form

$$X_{t_k} = X_{t_{k-1}} \exp\left(\frac{-\alpha}{365}\right) + \sigma\sqrt{\frac{1-\exp\left(\frac{-2\alpha}{365}\right)}{2\alpha}} N(0,1) + \sum_{i=1}^{N_{1/365}} Z_i, \qquad (7)$$

where $N(0,1)$ is a standard normally distributed variable, $N_{1/365}$ is a Poisson random variable with the intensity parameter $\frac{\lambda}{365}$, Z_i are mixed-exponentially distributed random variables. A sample trajectory put on the seasonality is shown in Figure 6.

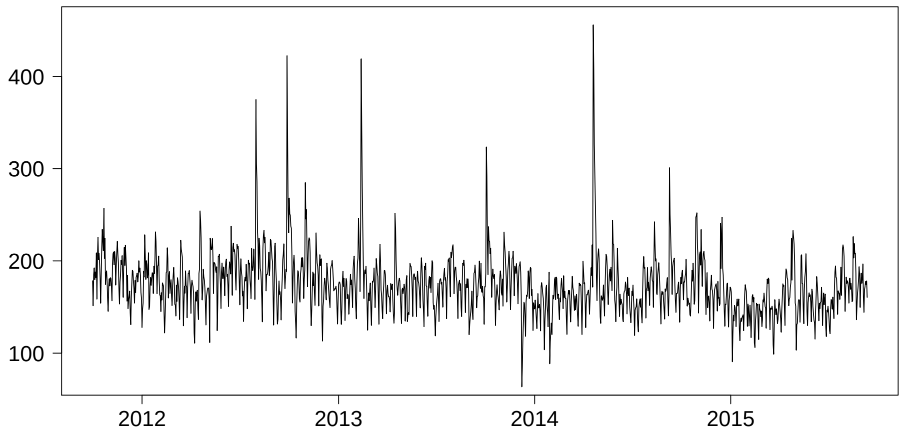

Fig. 6. Simulated sample path in PLN/MWh

7.2 Goodness of fit of the sample paths

The comparison of two moments and 15%, 85% quantiles of the historical log-returns and log-returns of 5000 simulated trajectories is shown in Table 2.

The Kolmogorov-Smirnov test for the equality of distributions of the real log-increases and the log-increases of the simulated data gives no evidence to reject the null hypothesis of the equality of these distributions at a reasonable level – the averaged p-value (over 5000 samples) is equal to 0.09.

The reestimation procedure was also conducted, i.e. for each simulated path all the parameters were estimated and then were averaged over samples – the resulting parameters' values were very similar to those computed during the estimation described in Section 5.

	mean	st. dev.	15% quantile	85% quantile
real data	-0.00023	0.153	-0.126	0.131
simulations	-0.00008	0.14	-0.128	0.133

Table 2. Moments and quantiles of the historical and 5000 simulated log-returns (averages)

8 Analytical formula for the forward price

One of the biggest advantages of the model is that it enables to derive an analytical formula

$$F(t,T) = \mathbb{E}^{\mathbb{Q}}[S_T|S_t] \tag{8}$$

for the forward prices F(t,T), $0 < t \leq T$, $T \in \mathcal{T}$, where $\mathbb{Q}$ is an equivalent risk-neutral measure. Thanks to this analytical formula one can create a forward curve and thanks to the change of measure it is possible to make the model suited to the actual prices. From mathematical point of view, there are uncountably many equivalent, potentially risk-neutral measures and the task is to pin down the appropriate one. From financial point of view, the considered electrical energy market is incomplete, as there are more sources of randomness (hence risk) than risky assets, thus not every payoff may be replicated (hedged) with this underlying asset and not risky one, for instance a bank account or a bond. In the model there are two sources of risk: diffusion risk connected to the Wiener process and jump risk related to the compound Poisson process. To deal with the problem of calibration of the model to the actual market situation and quoted forward contracts, the notions of the market prices of diffusion risk and jump risk are introduced. Ascribing the concrete numerical values to the parameters which denote the market prices of risks uniquely determines the choice of the appropriate risk-neutral measure. For details, we refer the reader to [14].

Theorem 1. *The analytical formula for the forward price within the model defined in Section 2 by (2) and (3) is equal to*

$$\begin{aligned} F(t,T) &= \mathbb{E}^{\mathbb{Q}}[S_T|\mathcal{F}_t] = \\ & G(T)\left(\frac{S_t}{e^{g(t)}}\right)^{e^{-\alpha(T-t)}} \exp\left(\int_t^T \sigma e^{-\alpha(T-s)}\left(\frac{1}{2}\sigma e^{-\alpha(T-s)} - \theta^{\mathbb{Q}}\right) ds\right) \cdot \\ & \exp\left(\int_t^T \left(e^{m_d} q_d \sum_{i=1}^m q_i \frac{\xi_i e^{\alpha(T-s)}}{\xi_i e^{\alpha(T-s)}+1} + e^{m_u} p_u \sum_{j=1}^n p_j \frac{\eta_j e^{\alpha(T-s)}}{\eta_j e^{\alpha(T-s)}-1}\right)\lambda^{\mathbb{Q}} ds \right. \\ & \left. - \lambda^{\mathbb{Q}}(T-t)\right), \end{aligned} \tag{9}$$

where $\theta^{\mathbb{Q}}$ is the market price of diffusion risk and $\frac{\lambda}{\lambda^{\mathbb{Q}}}$ is the market price of jump risk with $\lambda^{\mathbb{Q}}$ an intensity of the compound Poisson process after change of measure to the risk-neutral $\mathbb{Q}$.

Proof. We refer the reader to [14].

9 Valuing options on electricity spot price

A call option contract on the underlying asset, which in this case is electricity spot price, gives its holder at expiration date T the right (it is not an obligation as in case of a forward contract) to buy electricity for K instead of S_T. Likewise, a put option contract secures the right to sell electricity for K instead of S_T.

The problem of pricing at time t a call vanilla option $C_{t,T}(K)$ on electicity spot, expiring at time T and with strike K, is equivalent to finding value of the following expression

$$\begin{aligned} C_{t,T}(K) = \exp(-r(T-t))\mathbb{E}^{\mathbb{Q}}[\max(S_T - K, 0)|S_t] = \\ \exp(-r(T-t))\mathbb{E}^{\mathbb{Q}}[\max(\exp(X_T + g(T)) - K, 0)|X_t], \end{aligned} \tag{10}$$

where r is a discount rate. Analogously, a price of a put vanilla option $P_{t,T}(K)$ is given by

$$P_{t,T}(K) = \exp(-r(T-t))\mathbb{E}^{\mathbb{Q}}\left[\max(K - \exp(X_T + g(T)), 0)|X_t\right]. \tag{11}$$

Here we assume that $\mathbb{Q} = \mathbb{P}$, i.e. that we price options with respect to the physical measure, where the probabilities of events are induced by the historical realisation of prices values. It is due to the fact that methodology and results of the calibration of the model to the risk-neutral measure $\mathbb{Q}$ lie out of the scope of this article, and are the subject of a forthcoming paper. In this section we concentrate on a form of the price estimator and application of the driving process simulation method described earlier.

An adequate tool to cope with such defined problem is a Monte Carlo setup. By the strong law of large numbers, Monte Carlo estimators of (10) and (11) are equal to

$$\widehat{C}_{t,T} = \exp(-r(T-t))\frac{1}{n}\sum_{i=1}^{n}\max\left(\exp\left(X_T^{(i)} + g(T)\right) - K, 0\right) \tag{12}$$

and

$$\widehat{P}_{t,T} = \exp(-r(T-t))\frac{1}{n}\sum_{i=1}^{n}\max\left(K - \exp\left(X_T^{(i)} + g(T)\right), 0\right), \tag{13}$$

where $X_T^{(1)}, X_T^{(2)}, \ldots, X_T^{(n)}$ are sample values of the process X at time T obtained by simulating trajectories from t up to T according to the formula (7).

In Table 3 there are presented call and put option prices on electricity spot with time to expiry equal to 90 days, i.e. $T - t = \frac{90}{365}$, and with different strike prices K. The value of the seasonality function at expiry date is $\exp(g(T)) = 158$ PLN/MWh, discount rate $r = 0.02$.

K	120	130	140	150	160	170	180	190
$\widehat{C}_{t,T}$	43.90	34.11	24.66	16.24	9.89	6.00	3.85	2.69
$\widehat{P}_{t,T}$	0.14	0.30	0.80	2.33	5.93	11.00	19.79	28.59

Table 3. Call and put option prices on electricity spot with time to expiry equal to 90 days and the seasonality function at expiry date equal to 158 PLN/MWh

10 Conclusions

In the article the authors introduce the new model for electricity spot prices which are quoted on the Polish Power Exchange, taking into account all the specificity of the Warsaw market, as well as the electrical energy prices specificity in general. Several novel ideas concerning seasonality matching, spikes filtering, jump-size distribution, etc. are put into practice. The parameters are estimated basing on the historical data. The model is validated by performing simulations and tests for goodness of fit, which legitimize the proposed approach. Within the model there exists an analytical formula for the forward prices allowing for convenient calibration of the model to the forward contracts quoted on the exchange, making use of the notions of the market prices of diffusion and jump risks. Finally, valuing of options on electricity spot price is performed.

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