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A GENERALIZATION
OF PETRI NETS
BY EQUIPPING THEM
WITH INPUTS AND OUTPUTS

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A b s t r a c t . S t r e s z c z e n i e

The concept of Petri nets is generalized by allowing inputs for absorbing information and outputs for emitting information. For the structures thus obtained, called seminets, a concept of behaviour similar to that for Petri nets is introduced formally. A partial operation of composing seminets by combining outputs with inputs is introduced such that the behaviour of a seminet which consists of seminets with known behaviours can be obtained by combining these behaviours.

UOGÓLNIENIE SIECI PETRI
PRZEZ WYPOSAŻENIE ICH W WEJŚCIA I WYJŚCIA

Praca prezentuje uogólnienie sieci Petri polegające na dopuszczeniu wejść do przyjmowania informacji i wyjść do wysyłania informacji. Dla otrzymywanych w ten sposób struktur, zwanych półsieciami, wprowadzono pojęcie zachowania się. Zaproponowano częściową operację składania półsieci przez łączenie wyjść z wejściami. Operacja ta ma tę cechę, że zachowanie się półsieci która składa się z półsieci o znanych działaniach można otrzymać przez odpowiednie złożenie tych ostatnich.

1. INTRODUCTION

Petri nets represent concurrent systems in terms of memory-like objects (places capable of storing tokens), action-like objects (transitions absorbing and emitting tokens), and relations between such objects (directed links along which tokens may flow between places and transitions). However, they do not offer any convenient means for representing how a system communicates with the external world.

In this paper the concept of Petri nets is extended in a manner which allows to represent explicitly also the communication of a system with the external world. This is done by introducing special incoming and outgoing links for representing the exchange of information (tokens) between a system and its environment. Thus we come to structures of a new type, called seminets, which have not only links between places and transitions, but also links without a source object (dangling incoming links) or without a target object (dangling outgoing links).

The behaviours of seminets are described in terms of unfoldings similar to those defined for Petri nets in several forms (traces [M77], processes [GLT80], [W80], [GR83], [BD87], pomsets [M88], and net computations [DMM89]). Unfoldings of a seminet are seminets which represent particular ways of executing transitions of this seminet or, equivalently, particular flows of tokens. For simplicity we consider only finite unfoldings and represent the behaviour of a seminet by the universe of its possible finite unfoldings (without a loss of generality we may assume that such a universe is a set). This way of representing the behaviours extends easily on seminets whose tokens contain information and whose transitions transform such information (interpreted seminets). It is different from a possible representation by event structures as in [Wi86] for Petri nets, but not less informative.

An important property of seminets is that by dividing the set of places and transitions into two disjoint subsets and by splitting all links between elements of one subset and elements of the other one can decompose a seminet into parts which are seminets. Put in another way, a seminet may consist of two seminets in the sense that its places, transitions, and links coincide with those of the components, the common links of the

components being outgoing in one component and incoming in the
other, or vice-versa. This leads to a partial operation of
composing seminets by combining dangling links.

By composing unfoldings of given seminets one can construct
unfoldings of their composition. Such a construction reflects the
exchange of tokens between the given seminets along their common
links. It results in a compositionality of behaviours in the
sense that the behaviour of a seminet consisting of some seminets
can be obtained by combining the behaviours of components.

The paper consists of a part about the concept of seminets,
followed by a part about behaviours of seminets, and by a part
about compositionality. The presented ideas and results are
improved versions and consequences of ones due to earlier works
(cf. [W88], [MW88], [W89], and MW00]).

2. SEMINETS

The concept of a seminet generalizes that of a Petri net. Like
Petri nets, a seminet has nodes of two types and links, some
pairs of nodes of different types connected by links. However, in
contrast to Petri nets where each link connects two nodes, in
seminets there may be links without a source or a target node
(dangling links). Without a loss of generality we may restrict
ourselves to seminets with places of infinite capacity.

2.1. DEFINITIONS

A *seminet* is $M = (P_M, T_M, Z_M, in_M, out_M)$, where
(1) P_M, T_M, Z_M are finite, mutually disjoint sets (of *places*,
transitions, and *links*, respectively),
(2) in_M and out_M are functions asigning to each $x \in P_M \cup T_M$
respectively a subset $in_M(x) \subseteq Z_M$ (of *incoming* links) and
a subset $out_M(x) \subseteq Z_M$ (of *outgoing* links) such that:
   (2.1) for each $z \in Z_M$ there exist at most one $x \in P_M \cup T_M$
        such that $z \in out_M(x)$ (a *source* of z, written as
        $source_M(z)$) and at most one $y \in P_M \cup T_M$ such that
        $z \in in_M(y)$ (a *target* of z, written as $target_M(z)$),
   (2.2) for each $z \in Z_M$, $source_M(z)$ and $target_M(z)$ do not
        belong simultaneously to P_M or to T_M.
A *labelling* of M is a function λ assigning to each
$x \in P_M \cup T_M \cup Z_M$ an element $\lambda(x)$ of a given set (a *label*).
An *interpretation* of M is a labelling λ such that each $\lambda(p)$ with

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$p \in P_M$ is a set, $\lambda(z) = \lambda(p)$ for each $z \in in_M(p) \cup out_M(p)$ with $p \in P_M$, and each $\lambda(t)$ with $t \in T_M$ is a set of functions, each function assigning to each $z \in in_M(t) \cup out_M(t)$ some element of $\lambda(z)$. A *labelled* (resp.: *interpreted*) seminet is a pair $\alpha = (M,\lambda)$, where M is a seminet and λ is a labelling (resp.: an interpretation) of M. A *marking* of a seminet M (resp.: of an interpreted seminet $\alpha = (M,\lambda)$) is a function μ assigning to each $p \in P_M$ a number $\mu(p) \in \omega$ (= $\langle 0,1,\ldots\rangle$) of *tokens* (resp.: a multiset $\mu(p)$ of elements of $\lambda(p)$, each element representing the *content* of a token). A *marked* seminet (resp.: a marked interpreted seminet) is a pair $\sigma = (S,\mu)$, where S is a seminet (resp.: an interpreted seminet) and μ is a marking of S. A *subseminet* of a seminet S is a restriction of S to a subset of elements (places, transitions, and links) such that the incoming and outgoing links of each place and each transition are preserved. A seminet M is said to be *isomorphic* to a seminet N iff there exists a bijection (an *isomorphism*)
$b: P_M \cup T_M \cup Z_M \longrightarrow P_N \cup T_N \cup Z_N$ such that $b(P_M) = P_N$, $b(T_M) = T_N$, $b(Z_M) = Z_N$, and $b(in_M(x)) = in_N(b(x))$, $b(out_M(x)) = out_N(b(x))$ for all $x \in P_M \cup T_M$. A labelled (resp.: interpreted, marked, interpreted and marked) seminet is said to be isomorphic to another such a seminet iff the corresponding usual seminets are isomorphic with an isomorphism which preserves the labelling (resp.: the interpretation, the marking, the interpretation and the marking). Sometimes it is convenient to think of seminets up to isomorphism, that is to deal with isomorphism classes of seminets, or *abstract* seminets, rather than with concrete seminets. ▌

2.2. EXAMPLES

In fig. 2.1 we show a seminet A with $P_A = \langle p\rangle$, $T_A = \langle t\rangle$, $Z_A = \langle a,b,c,d,e,f\rangle$, $in_A(p) = \langle a,b\rangle$, $in_A(t) = \langle c,e\rangle$, $out_A(p) = \langle c,d\rangle$, $out_A(t) = \langle a,f\rangle$.
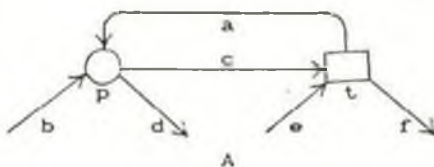

Fig. 2.1

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In fig.2.2 we show marked seminets $\alpha = (A,\mu_A)$, $\beta = (B,\mu_B)$, $\gamma = (C,\mu_C)$ with A as before, a marking μ_A of A defined by $\mu_A(p) = 2$, $P_B = \langle p \rangle$, $T_B = \emptyset$, $Z_B = \langle a,b,c,d \rangle$, $in_B(p) = \langle a,b \rangle$, $out_B(p) = \langle c,d \rangle$, $\mu_B(p) = 2$, $P_C = \emptyset$, $T_C = \langle t \rangle$, $Z_C = \langle a,c,e,f \rangle$, $in_C(t) = \langle c,e \rangle$, $out_C(t) = \langle a,f \rangle$, $\mu_C = \emptyset$. The seminets B and C are subseminets of A.
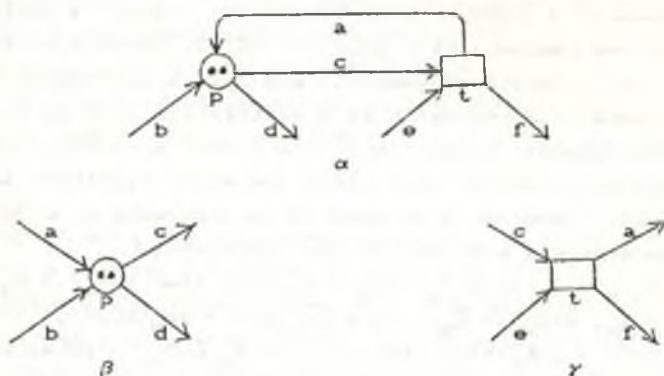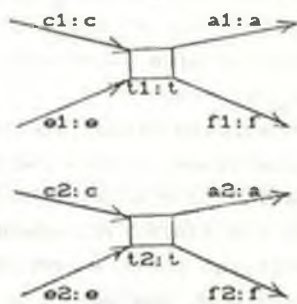

Fig. 2.2

In fig.2.3 we shaw labelled seminets δ, ϕ, ψ. Each element x with a label ζ is represented as $x{:}\zeta$. When considered up to isomorphism such seminets can be viewed as seminets with anonymous elements and represented as in fig.2.4.
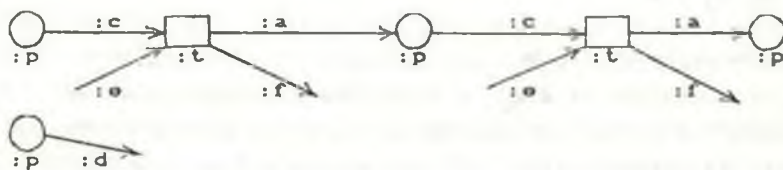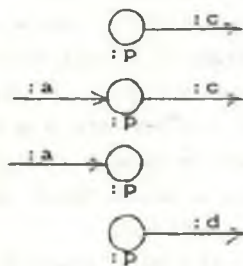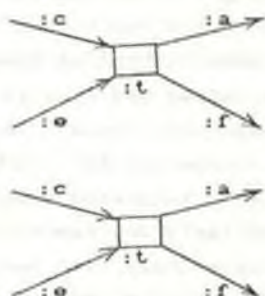
Fig. 2.3

Fig. 2.4

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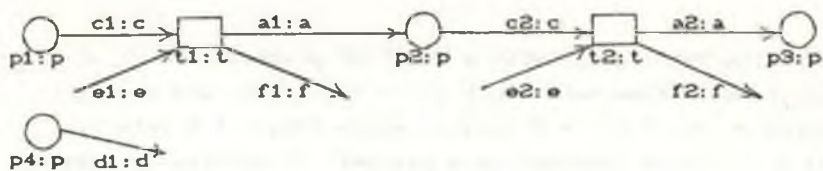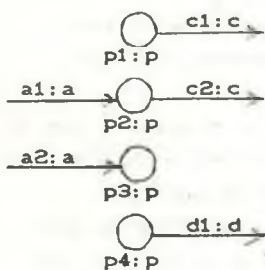
A finite Petri net N with a set P of places, a set T of
transitions, a flow relation $F \subseteq P \times T \cup T \times P$, and a weight
function $W: P \times T \cup T \times P \longrightarrow \omega$, where $W(u,v) > 0$ only for
$(u,v) \in F$, can be regarded as a seminet. It suffices to assume
$P_N = P$, $T_N = T$, $Z_N = \langle (u,i,v): (u,v) \in F, i \in \langle 1,\ldots,W(u,v) \rangle \rangle$,
and define in_N and out_N by $in_N(x) = \langle (u,i,v) \in Z_N: v = x \rangle$ and
$out_N(x) = \langle (u,i,v) \in Z_N: u = x \rangle$. Here each link is a triple
$z = (u,i,v)$ with $(u,v) \in F$, $1 \leq i \leq W(u,v)$, $source_N(z) = u$,
$target_N(z) = v$.

A Predicate/Transition net N with a set P of places, a set T
of transitions, a flow relation $F \subseteq P \times T \cup T \times P$, each $p \in P$ of
some arity(p) $\in \omega$, each $(u,v) \in F$ endowed with a multiset
$W(u,v) = \Sigma\, k(u,v,\tau)\tau$, where each τ is an n-tuple of terms with
$n = $ arity(q) for the corresponding place (q = u or q = v) and
$k(u,v,\tau) \in \omega$, and each $t \in T$ is endowed with a first-order
formula $C(t)$, where free variables of $C(t)$ occur in the multisets
$W(u,v)$ with $u = t$ or $v = t$, can be regarded as an interpreted
seminet. The underlying seminet can be defined by assuming
$P_N = P$, $T_N = T$,
$Z_N = \langle (u,i,\tau,v): (u,v) \in F, k(u,v,\tau) > 0, i \in \langle 1,\ldots,k(u,v,\tau) \rangle \rangle$,
$in_N(x) = \langle (u,i,\tau,v) \in Z_N: v = x \rangle$, and
$out_N(x) = \langle (u,i,\tau,v) \in Z_N: u = x \rangle$. The interpretation in
a domain D with the corresponding functions and relations can be
defined by assuming $\lambda(p) = D^n$ for each $p \in P$ with arity(p) = n,
$\lambda(z) = \lambda(p)$ for each $z \in in_N(p) \cup out_N(p)$ and $p \in P$, and by
defining $\lambda(t)$ as the set of correspondences f_σ between links
$z = (u,i,\tau,v)$ such that $u = t$ or $v = t$ and tuples $\tau(\sigma)$, where σ
is a substitution of elements of D for variables such that $C(t)$
is satisfied and $\tau(\sigma)$ is the result of this substitution in the
corresponding tuple τ. In the case of arity(p) = 1 for all $p \in P$,
$C(t) = $ true for all $t \in T$, and of a one-element domain $D = \langle d_0 \rangle$,
such an interpreted seminet can be regarded as a usual Petri net.

Similarly one can represent parts of Petri nets and
Predicate/Transition nets such that together with each place or
transition also the corresponding adjacent links belong to the
part under consideration. For example, the part P in fig.2.5 of
a Predicate/Transition net, where $G: X \times S \longrightarrow S$, $H: X \times S \longrightarrow Y$,
$v \in S$, can be regarded as a marked interpreted seminet $\sigma = (\alpha,\mu)$
with $\alpha = (A,\lambda)$, A as in fig.2.1, $\lambda(e) = X$, $\lambda(f) = Y$,

$\lambda(p) = \lambda(a) = \lambda(b) = \lambda(c) = \lambda(d) = S$, and

$\lambda(t) = \langle\langle (e,u),(c,v),(a,G(u,v)),(f,H(u,v))\rangle\rangle$: $u \in X$, $v \in S\rangle$. █
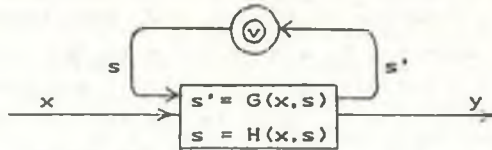
P

Fig. 2.5

3. BEHAVIOURS OF SEMINETS

Seminets represent concurrent systems and their behaviours.
The behaviour of a concurrent system is represented by possible
unfoldings of the corresponding seminet.

As in the case of Petri nets, an unfolding of a seminet is
a labelled, acyclic, choice-free seminet that consists of
occurrences of elements of the given seminet, each occurrence
endowed with some information about the occurring element.

3.1. DEFINITIONS

An *unfolding* of a seminet M (resp.: of an interpreted seminet
$\alpha = (M,\lambda)$) is a labelled seminet $\phi = (K,x)$, where
(1) the seminet K is *acyclic* and *choice-free* in the following
    sense:
    (1.1) the reflexive and transitive closure of the relation
           defined by $(u,v) \in R$ iff $u \in in_K(v)$ or $v \in out_K(u)$ is a
           partial order on $P_K \cup T_K \cup Z_K$ (the *causal order* of K).
    (1.2) for each $p \in P_K$, $in_K(p)$ and $out_K(p)$ are at most
           one-element sets.
(2) the labelling x is a mapping
    f: $P_K \cup T_K \cup Z_K \longrightarrow P_M \cup T_M \cup Z_M$ such that:
    (2.1) $f(P_K) \subseteq P_M$, $f(T_K) \subseteq T_M$, $f(Z_K) \subseteq Z_M$.
    (2.2) $f(in_K(p)) \subseteq in_M(f(p))$ and $f(out_K(p)) \subseteq out_M(f(p))$ for
           all $p \in P_K$.
    (2.3) f is one-to-one on $in_K(t) \cup out_K(t)$ and such that
           $f(in_K(t)) = in_M(f(t))$ and $f(out_K(t)) = out_M(f(t))$ for
           all $t \in T_K$.
    (2.4) for each $z \in Z_K$,
           $source_M(f(z))$ is defined iff $source_K(z)$ is defined

and then $f(\text{source}_K(z)) = \text{source}_M(f(z))$;

similarly for the target

(resp.: the labelling $\varkappa$ is a pair (f,g), where f is as before and g is a function on $P_K \cup T_K \cup Z_K$ such that:

(2.5) for each $p \in P_K$ and each $z \in \text{in}_K(p) \cup \text{out}_K(p)$,

$g(p) \in \lambda(f(p))$, $g(z) \in \lambda(f(z))$, and $g(z) = g(p)$,

(2.6) for each $t \in T_K$, $g(t)$ is the function assigning to each $z \in \text{in}_M(f(t)) \cup \text{out}_M(f(t))$ the element $g(f^{-1}(z))$ of $\lambda(z)$ and it belongs to $\lambda(f(t))$).

Given a marking μ of M (resp.: of α), we say that ϕ *applies* to μ with a marking ν as a *result* iff for each $p \in P_M$ the set $\langle x \in P_K : f(x) = p, \text{in}_K(x) = \emptyset \rangle$ has a cardinality $\phi^-(p)$ (resp.: is an instance of a multiset $\phi^-(p)$) such that $\phi^-(p) \leq \mu(p)$ and $\nu(p) = \mu(p) - \phi^-(p) + \phi^+(p)$, where $\phi^+(p)$ is the cardinality of the set $\langle x \in P_K : f(x) = p, \text{out}_K(x) = \emptyset \rangle$ (resp.: the multiset with the instance $\langle x \in P_K : f(x) = p, \text{out}_K(x) = \emptyset \rangle$). The universe of such unfoldings is written as $\text{unf}(M,\mu)$ (resp.: as $\text{UNF}(\alpha,\mu)$). For a marked seminet $\sigma = (M,\mu)$ (resp.: a marked interpreted seminet $\sigma = (\alpha,\mu)$) we define $\text{unf}(\sigma)$ as $\text{unf}(M,\mu)$ (resp.: $\text{UNF}(\sigma)$ as $\text{UNF}(\alpha,\mu)$) and by $\text{unf}(M)$ (resp.: by $\text{UNF}(\alpha)$) we denote the union of $\text{unf}(M,\mu)$ (resp.: of $\text{UNF}(\alpha,\mu)$) over μ. Finally, by unf (resp.: by UNF) we denote the universe of all possible unfoldings of seminets (resp.: of interpreted seminets). ▮

3.2. EXAMPLES

The labelled seminet ϕ in fig.2.3 (and its abstract variant $[\phi]$ in fig 2.4) is an unfolding of the marked seminet β in fig.2.2. This unfolding represents emitting two tokens which are present in p along links c and d, respectively, receiving a token along link a, and receiving a token along link a followed by emitting this token along link c. The labelled seminet ψ in fig.2.3 (and its abstract variant $[\psi]$ in fig.2.4) is an unfolding of the marked seminet γ in fig.2.2. This unfolding represents two independent executions of t, each execution consisting of coincident events of receiving tokens along links c and e and emitting tokens along links a and f. The labelled seminet δ in fig.2.3 (and its abstract variant $[\delta]$ in fig.2.4) is an unfolding of the marked seminet α (and of its underlying unmarked seminet A in fig.2.1). This unfolding represents a behaviour of α which is a combination of those represented by ϕ and ψ.

The labelled seminet π in fig.3.1 is an unfolding of the
interpreted marked seminet corresponding to P in fig.2.5. This
unfolding represents two consecutive executions of the transition
of P, the second execution using information produced by the
first one. In this case the label of each element consists not
only of the respective element of the unfolded seminet, but also
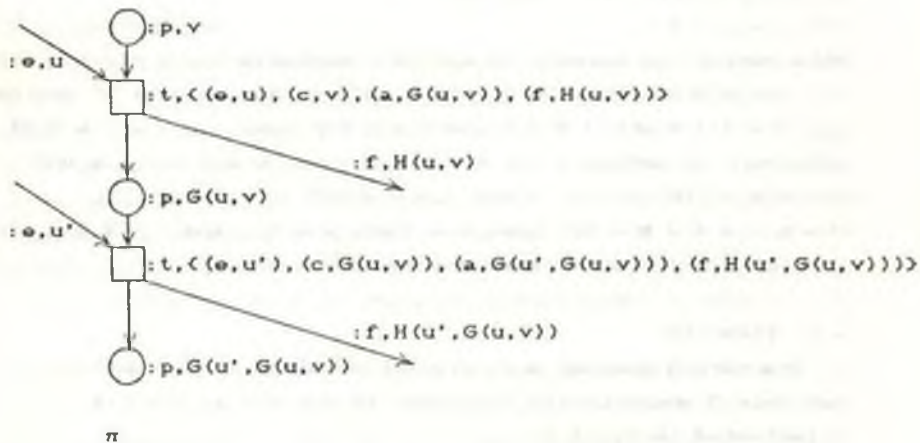of the information about the involved data and how these data are
transformed. ▌

π

Fig. 3.1

4. COMPOSITIONALITY

By splitting links which connect pairs of nodes into pairs of
dangling links one can decompose a seminet into subseminets.
The same can be done for the respective unfoldings. We shall see
that decompositions of seminets and those of unfoldings are
related such that the behaviour of a seminet which consists of
some given seminets can be obtained from the behaviours of the
component seminets.

4.1. DEFINITIONS

A seminet A (resp.: a labelled, or interpreted, or marked
seminet $\alpha = (A,\zeta)$, or a marked interpreted seminet $\sigma = (\alpha,\mu)$)
is said to *consist* (or to be *composed*) of seminets B and C
(resp.: labelled, or interpreted, or marked seminets $\beta = (B,\eta)$
and $\gamma = (C,\zeta)$, or marked interpreted seminets $\tau = (\beta,\nu)$ and
$\rho = (\gamma,\pi)$), written as A = B + C (resp. : as $\alpha = \beta + \gamma$, or

$\sigma = \tau + \rho$), iff the following conditions are fulfilled:

(1) $P_A = P_B \cup P_C$ with $P_B \cap P_C = \emptyset$,

(2) $T_A = T_B \cup T_C$ with $T_B \cap T_C = \emptyset$,

(3) $Z_A = Z_B \cup Z_C$ with $z \in Z_B \cap Z_C$ only if

either $source_A(z) \in P_B \cup T_B$ and $target_A(z) \in P_C \cup T_C$

or $source_A(z) \in P_C \cup T_C$ and $target_A(z) \in P_B \cup T_B$.

(4) $in_A = in_B \cup in_C$ and $out_A = out_B \cup out_C$.

(resp.: A,B,C fulfil (1) - (4) and

(5) $\zeta = \eta \cup \zeta$ with $\eta|Z_B \cap Z_C = \zeta|Z_B \cap Z_C$,

or α, β, γ fulfil (1) - (5) and

(6) $\mu = \nu \cup \pi$).

This definition extends on abstract seminets by: $u = v + w$ iff $u' = v' + w'$ for some $u' \in u$, $v' \in v$, $w' \in w$, and on sets of seminets by: $U = V + W$ iff $U = \langle u: u = v + w$ for some $v \in V$ and $w \in W \rangle$. Moreover, we define $u = v \oplus w$ iff $u = v + w$ and $u,v,w \in unf$, $u = v \& w$ iff $u = v + w$ and $u,v,w \in UNF$. $U = V \oplus W$ iff $U = \langle u: u = v \oplus w$ for some $v \in V$ and $w \in W \rangle$, and $U = V \& W$ iff $U = \langle u: u = v \& w$ for some $v \in V$ and $w \in W \rangle$. ∎

4.1. EXAMPLES

The marked seminet α in fig. 2.2 consists of the marked seminets β and γ in fig. 2.2, that is $\alpha = \beta + \gamma$. This is illustrated in fig. 4.1.
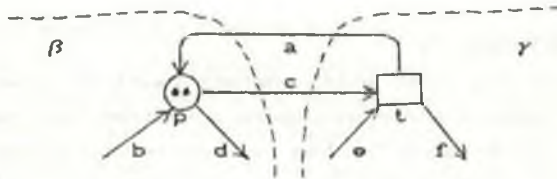

Fig. 4.1

The labelled seminet δ in fig. 2.3 consists of the labelled seminets ϕ and ψ in fig. 2.3, that is $\delta = \phi + \psi$. This is illustrated in fig. 4.2. As δ, ϕ, ψ are unfoldings of seminets, we have also $\delta = \phi \oplus \psi$. ∎
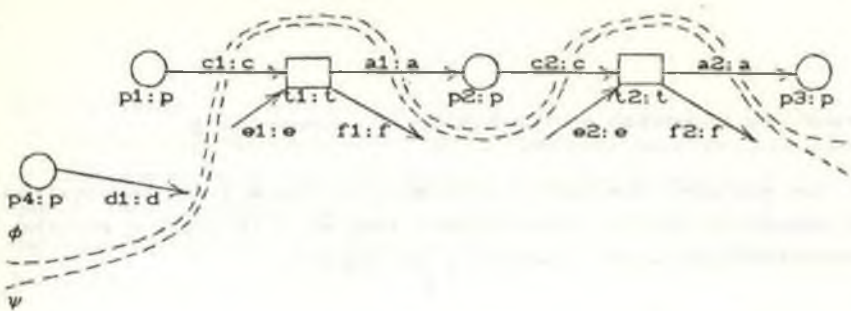
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Fig. 4.2

4.3. PROPOSITION

The relation "to consist of" is functional in the sense that
$A = B + C$ and $A' = B + C$ implies $A = A'$, and it extends on
abstract seminets in the sense that $A = B + C$ and $A' = B' + C'$ with
B' isomorphic to B and C' isomprphic to C implies that A is
isomorphic to A'. The respective partial operation of composing
seminets is commutative in the sense that $B + C = C + B$ whenever
either side is defined, and associative in the sense that
$(B + C) + D = B + (C + D)$ whenever either side is defined. ▌

Proof: The functionality is obvious. An isomorphism from A' to A
for $A = B + C$ and $A' = B' + C'$ with an isomorphism b from B' to B
and an isomorphism c from C' to C can be obtained by taking
$b \cup c$. The commutativity of the operation of composing seminets
is immediate. For associativity it suffices to notice that in the
case of seminets A,B,C,D,M such that $M = A + D$ and $A = B + C$ we
have $P_M = P_B \cup P_C \cup P_D$ with mutually disjoint P_B, P_C, P_D,
$T_M = T_B \cup T_C \cup T_D$ with mutually disjoint T_B, T_C, T_D, and
$Z_M = Z_B \cup Z_C \cup Z_D$ with $Z_B \cap Z_C \cap Z_D = \emptyset$. Hence $E = (P, T, Z, in, out)$
with $P = P_C \cup P_D$, $T = T_C \cup T_D$, $Z = Z_C \cup Z_D$, $in = in_C \cup in_D$,
$out = out_C \cup out_D$, is a seminet, $E = C + D$, and $M = B + E$, as
required. ▌

4.4. THEOREM

Each seminet can be obtained by composing atomic
(indecomposable) seminets, where an atomic seminet is either
a one-place seminet without transitions, or a one-transition
seminet without places, or a one-link seminet. Similarly for
labelled seminets, marked seminets, etc. ▌

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Proof: By induction on the number of elements. ▌

For example, the marked seminet α in fig. 2.2 can be obtained
by composing the one-place atomic seminet β in fig. 2.2 and the
one-transition atomic seminet γ in fig. 2.2.

4.5. PROPOSITION

A labelled seminet $\alpha = (A, \zeta)$ which consists of unfoldings
$\beta = (B, \eta)$ and $\gamma = (C, \zeta)$ of some seminets is an unfolding of a
seminet iff the following conditions are fulfilled:
(1) the transitive closure of the union of the causal order of B
    and the causal order of C is a partial order,
(2) A does not contain any dangling link which occurs both in B
    and in C, that is there is no $z \in Z_A$ with source or target
    undefined such that $\zeta(z) = \eta(u)$ for a link u of B and
    $\zeta(z) = \zeta(v)$ for a link v of C.
Similarly for a labelled seminet which consists of unfoldings of
interpreted seminets. ▌

Proof: Suppose that α is an unfolding which consists of β and γ.
Then (1) follows from the fact that the transitive closure of the
union of the causal order of B and the causal order of C
coincides with the causal order of A, and (2) follows from (2.4)
of 3.1. Conversely, suppose that α is a labelled seminet such
that $\alpha = \beta + \gamma$ and the conditions (1) and (2) are fulfilled. From
$A = B + C$ and (1) we obtain that A is acyclic and choice-free.
From $A = B + C$ and (2) we obtain that ζ satisfies a condition
corresponding to (2.4) of 3.1. Hence α is an unfolding of a
seminet which consists of some seminets with unfoldings β and γ,
respectively. The proof for interpreted seminets is similar. ▌

4.6. PROPOSITION

The operation on sets of unfoldings of seminets defined by
$(U, V) \longmapsto U \oplus V$ is commutative and associative. Similarly for
the operation on sets of unfoldings of interpreted seminets
defined by $(U, V) \longmapsto U \,\&\, V$. ▌

Proof: From 4.3. ▌

4.7. THEOREM

The correspondence M ⊢⟶ unf(M) between seminets and sets of their unfoldings is a *homomorphism* in the sense that unf(M + N) = unf(M) ⊕ unf(N) whenever the composition M + N is defined. Similarly for marked seminets, interpreted seminets, and marked interpreted seminets. ▮

Proof: Suppose that $\phi = (K,f)$ is an unfolding of M + N. Then the restrictions of ϕ to $f^{-1}(P_M \cup T_M \cup Z_M)$ and $f^{-1}(P_N \cup T_N \cup Z_N)$ are unfoldings ϕ_M and ϕ_N of M and N, respectively, and $\phi = \phi_M + \phi_N$. Hence unf(M + N) ⊆ unf(M) ⊕ unf(N). Suppose that $\phi_M = (A,g) \in$ unf(M) and $\phi_N = (B,h) \in$ unf(N) and that $\phi = \phi_M + \phi_N$. Then A + B is defined and $g|Z_A \cap Z_B = h|Z_A \cap Z_B$. Hence g ∪ h is a function and $\phi = (A + B, g \cup h)$. As $\phi \in$ unf(M) ⊕ unf(N), we obtain that ϕ in an unfolding. On the other hand, from the properties of g and h it follows that ϕ is an unfolding of M + N. Hence unf(M) ⊕ unf(N) ⊆ unf(M + N), which completes the proof for seminets. For interpreted seminets the proof is similar. ▮

5. CONCLUDING REMARKS

By allowing dangling links we have modified Petri nets and introduced structures called seminets. For such structures we have defined unfoldings and behaviours. We have shown that the correspondence between seminets and their behaviours is compositional in the sense that the behaviour of a composition of seminets with known behaviours can be obtained by combining these behaviours. This compositionality is similar to that in [M88] for Petri nets but perhaps more natural (the synchronization of sending and receiving of a token seems to be conceptually clearer than the synchronization of transitions), and more universal (it allows to deal easily with tokens containing information). Moreover, due to the fact that each seminet can be decomposed into atomic seminets, one can obtain the behaviour of a seminet by composing the easily definable behaviours of atomic seminets.

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